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Utilizing Leaky Lamb Waves in an Acoustic Waveguide for the Measurement of Liquid Properties

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Abstract:

A leaky Lamb wave sensor is based on the principle of multiple mode conversions in an acoustic waveguide. An initial ultrasonic Lamb wave propagates on a plate, but leaks energy to an adjacent liquid. Therein the exited wave propagates zigzag between the radiating and a second plate. At each reflection it partly reconverts, leading to a characteristic fingerprint in the received signals which has to be evaluated. Therefor a new method will be presented taking into account that all the liquid's state variables (sound velocity, density, temperature and viscosity) are influencing the received signal parameters. Additional aspects of this contribution are the parameterization of dispersion relations and the estimation of uncertainty in the measurement of sound velocities with the new method.

Key words: acoustic waveguide, dispersion, liquid properties, Leaky Lamb waves, ultrasound

Introduction

If an industrial process requires an evaluation of a liquid's composition or quality, ultrasonic sensors may be a good choice as they combine lots of features in one device: They have short reaction times, are precise, need low maintenance, are compact in size and don't have moving parts. In pipe solutions there is no need to change its cross section. Nevertheless it is possible to measure the concentration of components in solution, or to quantify the quality/ purity of a mixture. The key feature of ultrasonic waves is that they can be excited in such a way that they even make use of the sensors boundary. This could be the wall of the pipe or the plate of a purpose built ultrasonic sensor. The latter will be the focus of this contribution.

Leaky Lamb Waves in an Acoustic Waveguide

The presented Leaky Lamb wave sensor is based on the principle of multiple mode conversions in an acoustic waveguide (Fig. 1). An initial ultrasonic Lamb wave propagates on a solid plate, but partly leaks energy to an adjacent liquid. Therein the exited Leaky wave propagates zigzag between the radiating plate and a second plate which is spaced a few millimeters away parallel to the first plate. At each non specular reflection the Leaky wave partly reconverts to a Lamb wave which enables us to measure the arrival time and amplitudes of several wave packages on both plates [1].

During its propagation through the sensor the wave package has different velocities. In the plate it propagates with energy velocity c_{gr} whereas the angle for the radiation of the Leaky wave and its nonspecular reflection is mainly determined by the lesser phase velocity c_{ph}. As a consequence, a ray model for calculation of the liquid's sound velocity c_F has to take both, phase and energy velocity of the plate into account. The fundamental formula for these ray models is the definition of the Rayleigh angle

$$\theta_R = \arcsin\left(c_F / c_{ph}\right). \tag{1}$$

Therewith, the times of flight can be easily calculated by trigonometric considerations. Apart from that the amplitudes of the reconverted wave packages have to be determined. Therefor the reciprocity relation has to be formulated for the specific inhomogeneity of the radiated and reradiated waves. A good starting point for these calculations is in [2]. Following the ideas of Jia in [3] we could find analytical expressions for some amplitude ratios.

Dispersion relations

A basis for a proper sensor design as well as modeling of Lamb wave propagation and radiation of Leaky waves are the dispersion relations, that means frequency dependent functions for $c_{ph}(\omega)$, $c_{gr}(\omega)$, and for $\alpha_0(\omega)$ - the attenuation of the Leaky Lamb wave in the plate. A short derivation of the Leaky Lamb wave equation can be found in [4].

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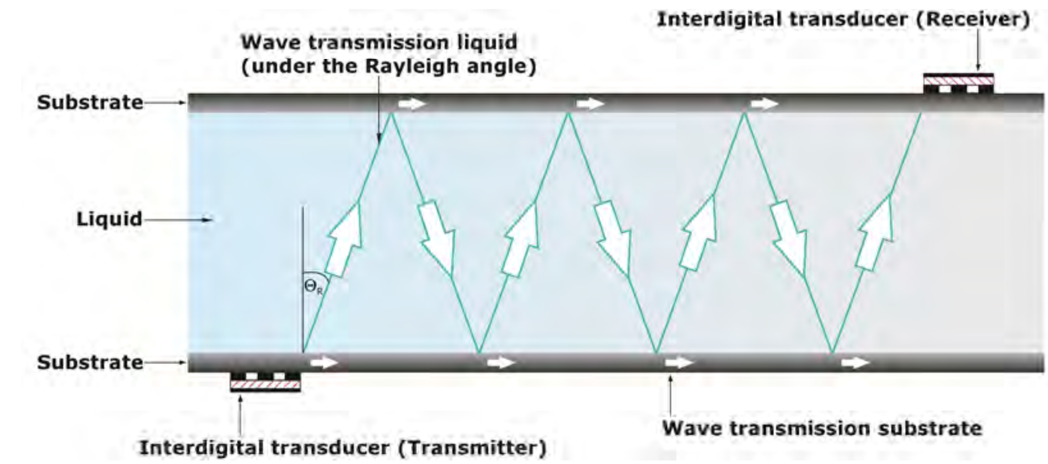

Fig. 1. Schematic of the waveguide sensor with zigzag propagation direction of the Leaky Lamb wave

Let k be the wave number, ω the angular frequency, and $2b$ the thickness of the plate (substrate), then the characteristic equation is

$$\alpha^2 K_\mathrm{m}^2 \tanh\alpha b \tanh\beta b = \left(\tanh\alpha b \tanh\beta b \cdot \alpha K_\mathrm{m} - 8\alpha\beta k^2 \tanh\alpha b + 2K_\mathrm{p}^2 \tanh\beta b\right) \cdot \left(\alpha K_\mathrm{m} - 8\alpha\beta k^2 \tanh\beta b + 2K_\mathrm{p}^2 \tanh\alpha b\right) \quad (2)$$

with

$$\alpha = \sqrt{k^2 - \left(\omega^2/c_\mathrm{L}^2\right)}, \quad \beta = \sqrt{k^2 - \left(\omega^2/c_\mathrm{T}^2\right)}$$

$$K_\mathrm{p} = \left(k^2 + \beta^2\right), \quad K_\mathrm{m} = -\left(k^2 - \beta^2\right) \cdot \frac{\omega}{\rho \cdot c_\mathrm{T}^2} \cdot Z_\mathrm{S}$$

Therein, ρ is the density of the plate, c_L and c_T are the sound velocities of the longitudinal and shear waves in the plate and Z_S is its radiation impedance. It is known e.g. from Pavlakovic in [5] and is

$$Z_\mathrm{S} = \frac{\rho_\mathrm{F} \cdot \omega}{\sqrt{\left(\omega^2/c_\mathrm{F}^2\right) - k^2}} = \frac{\rho_\mathrm{F} \cdot c_\mathrm{F}}{\sqrt{1 - \left(c_\mathrm{F}^2/c_\mathrm{Ph}^2\right)}} = \frac{\rho_\mathrm{F} \cdot c_\mathrm{F}}{\cos\theta_\mathrm{R}}. \quad (3)$$

Therwith both, the density ρ_F and the sound velocity c_F of the adjacent fluid influence the propagation of the Lamb waves.

It is possible to find pairs of ω and k that solve the characteristic equation (2). The phase velocity then is the ratio of these two quantities: $c_\mathrm{Ph} = \omega/k$. Fig. 2 shows the numerical determined dispersion curves for a 1.5 mm one-sided loaded steel plate at room temperature (Young's modulus $E = 196.24$ GPa, Poisson ratio $\nu = 0.3$, density $\rho = 8000\ \mathrm{kg/m^3}$).

With little modifications of (2) it is also possible to find energy velocities and attenuation factors, but the computational effort for finding the dispersion curves is mentionable high. This is why the authors suggest a parameterization of the numerical determined results.

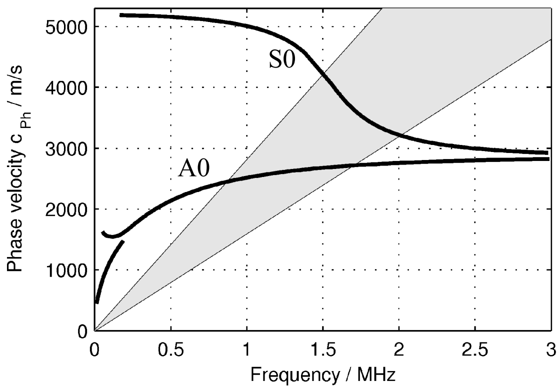

Fig. 2. Calculated dispersion curves (black lines) and region of mode excitation (gray shadowed zone) for a 1.5 mm one-sided loaded steel plate.

Therewith it is adequate to parameterize phase and energy velocity as well as damping of the Lamb wave as a function of frequency, the liquid properties, temperature and the sensor dimensions. The simplified model should be close to reality but also simple enough for microcontroller based computing. We compared the simplified model with exact calculations and found all parameterized values closer than 0.1% to the numerical determined values. Fig. 3 shows a histogram of the remaining sound velocity deviations over the entire temperature and liquid property range.

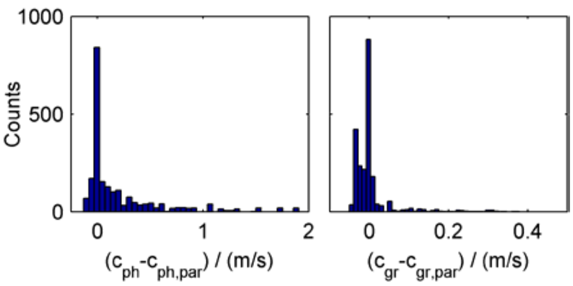

Fig. 3. Differences between calculated and parameterized phase (left) and group velocities (right).

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Measurement Setup and Signals

The measurements were made with a commercial available LiquidSens Probe Sensor (Fig. 4). For examination of temperature dependencies the whole setup was put into an oven, while the liquid under test (LUT) was stirred by a magnetic stirrer.

Fig. 4. Photograph of the acoustic waveguide sensor – here in a typical laboratory configuration.

The presented waveguide sensor is driven in such a way that mainly the antisymmetric mode (A0) is excited. This can be done by choosing an appropriate burst excitation (center frequency and duration, see activation region in Fig. 2). The results are received signals with well separated signal packages on each plate. Therewith it is possible to analyze liquids with sound velocities from 600 to 2000 m/s.

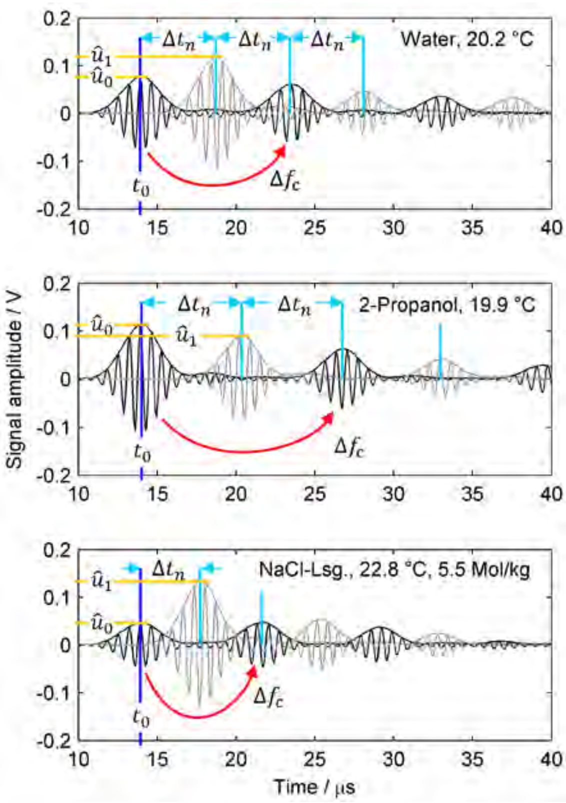

Fig. 5. Some characteristic signals and a selection of representative signal parameters that are part of the acoustic fingerprint. Black: Plate with transmission; Gray: On the opposite site of transmission.

The radiation angle θ_R as well as the propagation speeds, damping and coupling of the Lamb waves highly depend on the liquid properties to be measured. Therewith each liquid leaves a characteristic pattern in the received signals.

The Acoustic Fingerprint

The first step in the liquid property measurement is to determine characteristic signal parameters such as times-of-flight $(t_0, \Delta t_n)$, amplitudes $(\hat{u}_0, \hat{u}_1, \ldots)$ and center frequencies f_c (we denote this as the acoustic fingerprint). Fig. 6 shows some abstract fingerprints of selected liquids for different temperatures:

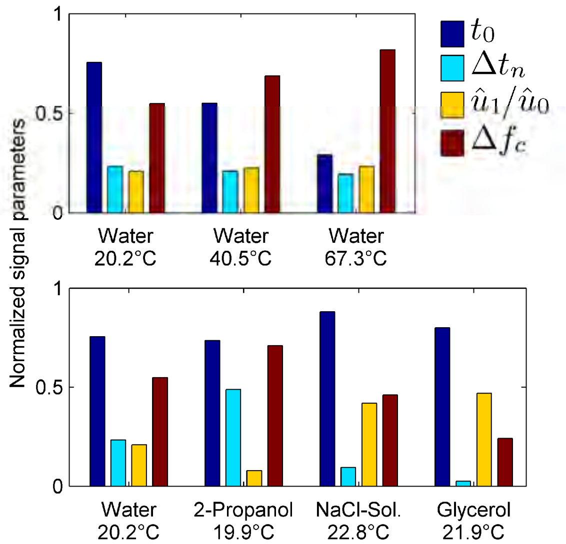

Fig. 6. Comparison of some acoustic fingerprints for different liquids at different temperatures.

It is mentionable that the fingerprint within a liquid does only change slightly with ambient conditions whereas fingerprints of different liquids are completely different.

The second step now is to read these acoustic fingerprints and translate them to the LUT's state variables which have significance for the process. These variables are the sound velocity c_F, the density ρ_F, the viscosity η_F and the liquid's temperature ϑ_F. Fig. 7 shows the extracted signal parameters in comparison to the most affine state variables.

The time of flight t_0 shows good correlation to the temperature ϑ_F if the liquid remains the same. The time-difference Δt_n seems to be inverse proportional to the sound velocity c_F whereas the amplitude ratio $\hat{u}_1/\hat{u}_0$ correlates with the specific acoustic impedance $Z_F = \rho_F c_F$ of the liquid. Finally, the frequency shift Δf_C is associated with the viscosity if the liquid remains the same.

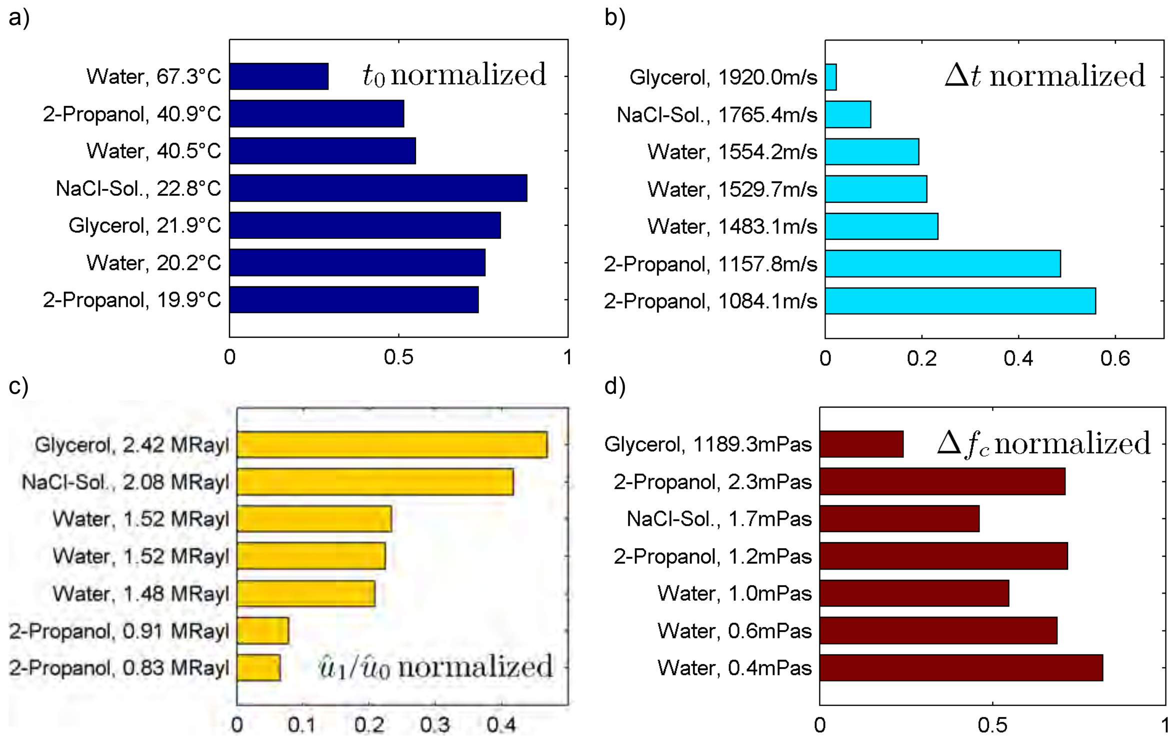

Fig. 7. Correlation of signal parameters with liquid properties: a) t_0 with temperature, b) Δt_n with sound velocity, c) $\hat{u}_1/\hat{u}_0$ with acoustic impedance and d) Δf_c with the viscosity of the liquid.

Discussion

The sound velocity and the specific acoustic impedance seem to have the best correlation to single selected signal parameters. This is why we started with simplified models for these two quantities and measured the temperature separately, knowing about its influence on the Lamb wave velocities:

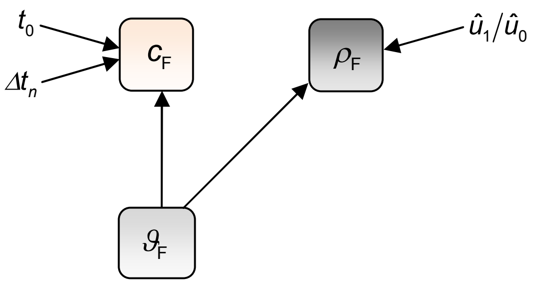

Fig. 8. First realization of the LiquidSens sensors – The liquid's temperature is measured separately.

As a result in special velocity-density-temperature combinations significant deviations of the sound velocity from the expected values could be observed. A complex calibration procedure can resolve this problem but with equation (2) and (3) we know about the interrelation between density, sound velocity and temperature via dispersion. Moreover, with the knowledge of Lamb wave attenuation there is additional influence of the liquid's viscosity. This is why the liquid's state should be described with the already mentioned state variables c_F, ρ_F, η_F and it's temperature ϑ_F - similar to the thermodynamic state variables. If the entire set of variables is determined, the dataset is consistent and the models can be used to calculate e.g. concentrations with much better accuracy. Fig. 9 visualizes the complete relationship between the state variables as well as the most affine signal parameters for each variable.

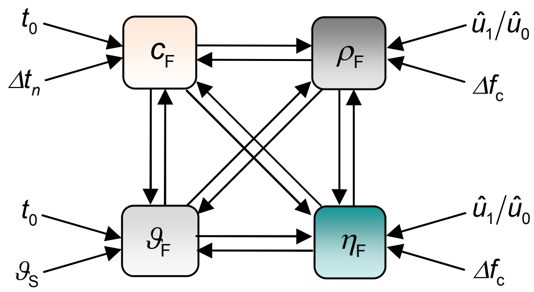

Fig. 9. Intercorrelation of the liquid's state parameters and signal parameters that have main influence in the models.

As a consequence of this new understanding the new generation of LiquidSens sensors makes use of every actual state variable in every calculation step. That means for instance, if the sound velocity has to be determined, the measured times of flight t_0 and Δt_n are used, but the predetermined values for ϑ_F, ρ_F and η_F are considered as well. Actually, this is implemented and tested for the determination of sound velocities and densities (Fig. 10).

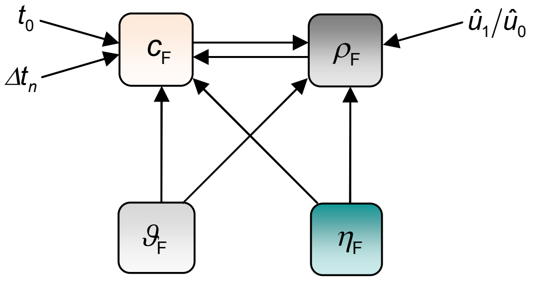

Fig. 10. Actual realization in the LiquidSens platform – The liquid's temperature as well as it's viscosity are not determined acoustically.

Example: Uncertainty of Liquid's Sound Velocity

Taking the new idea of cyclic parameter processing into account, it is a new challenge to make an estimate about uncertainty of measurement. On the one hand the inputs are intercorrelated. On the other hand there is no straight forward formula to be analyzed in the classical sense. The GUM, Supplement 1, suggests Monte Carlo simulations for such problems [9]. Therefore, each input variable must be considered uncertain with a characteristic probability density function (PDF). For simplicity all input values are considered to be Gaussian with a certain standard deviation. For the liquid's sound velocity the inputs are

- measured times of flight: $s_t = \pm 125$ ps
- measured temperature: $s_\vartheta = \pm 0.1$ K
- viscosity: unknown, i.e. fix
- calculated density: $s_\rho = \pm 25$ g/l

Therewith two cases, following Fig. 8 and following Fig. 10, were simulated for 3 different frequencies. In the first case only the times of flight were taken into account (Fig. 11) and the phase velocity is only adjusted by knowledge of c_{ph} in the mid frequency. The mid frequency f_{c2} shows the smallest systematic deviation whereas other frequencies may lead to deviations up to 3 %. Apart from that the uncertainty increases with the sound velocity of the liquid in general.

In the second case group and phase velocity were calculated with the parameterized model. The calculation of the fluid velocity was repeated several times for each simulated measurement until no significant change of the determined velocity could be noticed anymore. Fig. 12 shows the resulting uncertainties.

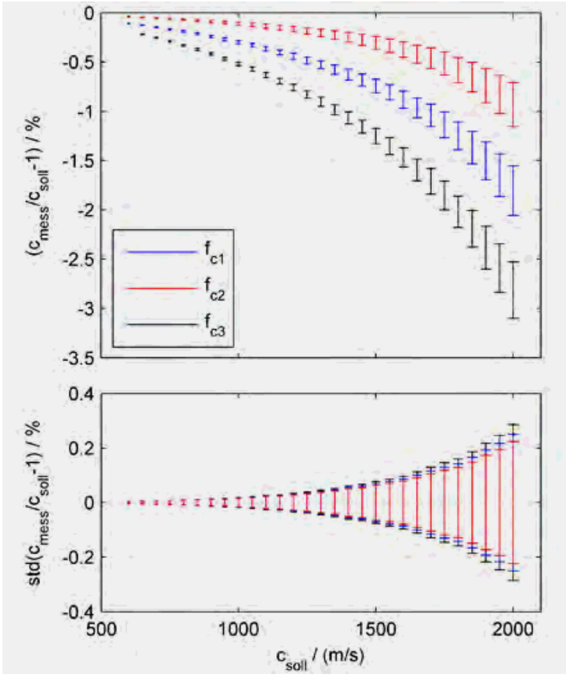

Fig. 11. Systematic (top) and nonsystematic (bottom) deviations between determined (index 'mess') and preset values (index 'soll') for three frequencies with $f_{c1} < f_{c2} < f_{c3}$ - 1st approach

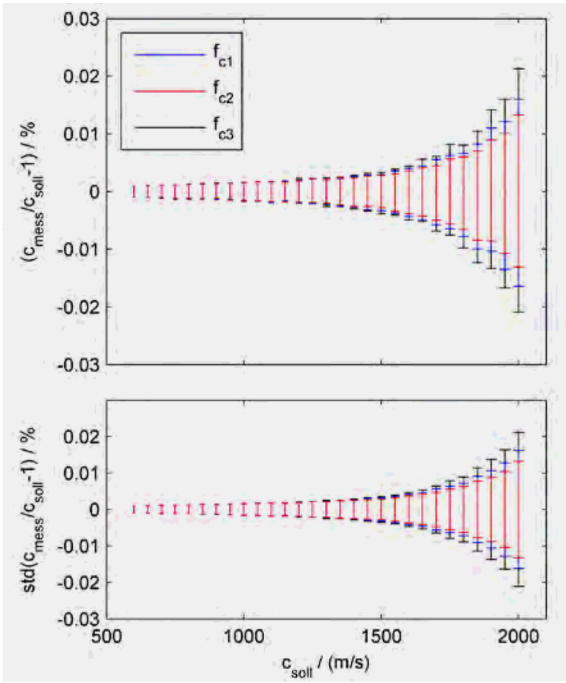

Fig. 12. Systematic (top) and nonsystematic (bottom) deviations between determined (index 'mess') and preset values (index 'soll') for three frequencies with $f_{c1} < f_{c2} < f_{c3}$ - cyclic determination

There is no systematic deviation and, although the uncertainty of the density was assumed to be very high, the change of other state variables does not cause high deviations in the sound velocity determination anymore. In comparison to the other method the uncertainty can be decreased by 1 order of magnitude, but the correlation between uncertainty and sound velocity magnitude seems to be characteristic for the measurement system.

Conclusion

The presented studies on an acoustic wave-guide have experimentally shown the complex correlations between signal parameters (acoustic fingerprint) and the liquid's state variables c_F, ρ_F, η_F and ϑ_F. It has further been demonstrated that there is an influence of all these state variables on the dispersion relations of the waveguide which again are essential for the calculation of all the liquid's state variables.

To overcome the dilemma of these cyclic dependencies, the dispersion relations were parameterized and included into a cyclic parameter processing. Based on a model of sound propagation as well as on this new scheme of parameter estimation Monte Carlo Simulations have shown that the new method will produce unbiased and to one magnitude order less uncertain results than another method without considering state dependent dispersion.

If the cyclic parameter estimation is fully implemented in the LiquidSens sensors, there is a good chance to improve the remaining uncertainties in both sound velocity and density measurement without time-consuming and expensive calibration procedures.

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