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Application of the Scaled Boundary Finite Element Method (SBFEM) for a numerical simulation of ultrasonic guided waves

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Abstract:
This paper addresses the computation of dispersion curves, mode shapes and propagation of elastic guided waves. It summarizes the approaches based on the Scaled Boundary Finite Element Method. Descriptions for plates, rods, pipelines and waveguides with an arbitrary cross section are included. The important steps for the approximation of the displacement in bounded and unbounded domains are stated. The grid generation process is explained. It is highlighted that the Scaled Boundary Finite Element Method is very efficient, if large portions of the domain are either straight or with a constant curvature. The computation of dispersion curves for layered structures is presented.

Key words: Ultrasonic guided waves, Non-Destructive Testing (NDT), Scaled Boundary Finite Element Method (SBFEM)

Introduction

Ultrasonic flaw detection is one of the most common methods used in Non-Destructive Testing. Pipes, plates, rods and other geometries can be tested with elastic guided waves. A guided wave can propagate if at least one direction of the structure is small in comparison to the wavelength. This wave type can travel over long distances. This allows a scan of the complete structure at once and leads to a fast inspection. To evaluate the results, a good understanding of the physics of wave propagation is necessary. The required information can be gained, e.g., with numerical simulations. However, most conventional finite element software is not well suited for this task. The high frequencies, and thus the ratio of wavelength and geometrical scale, demand a fine grid and small time steps for an accurate approximation. These factors lead to a time-consuming computation process. The long computation time can be significantly reduced using more sophisticated approaches. In this article, we present the Scaled Boundary Finite Element Method (SBFEM) as a suitable alternative to the conventional Finite Element Method (FEM) or the Finite Difference Method.

The inspection of a structure using guided waves starts with analyzing the dispersion properties. Dispersion describes the frequency dependence of the sound velocity. Assuming a dispersive wave propagation, two different types of velocities must be considered. On the one hand, there is the phase velocity which is directly linked with the wavelength, on the other hand, there is the group velocity describing the energy transportation speed. If one direction of the structure is long enough to be considered as infinite, these properties are usually summarized as dispersion curves. The curves build the foundation for many investigations in Non-Destructive Testing using guided waves.

The research on algorithms for computing dispersion curves has started with analytical solutions for isotropic materials in plates and rods which are infinite in one direction. Beside other approaches, semi-analytical methods can be used for analyzing the dispersion relations, for example the Semi-Analytical Finite Element (SAFE) and the Scaled Boundary Finite Element Method (SBFEM). These methods approximate the cross section of the sample by a finite element grid and solve the infinite direction analytically. In SAFE recent developments incorporated damping [1], however, only unbounded domains can be modeled. SAFE and SBFEM can also be used for simulating the wave propagation.

Another possibility to compute wave propagation and dispersion curves are higher order elements in a FEM or in a IsoGeometric Analysis [2]. Higher order elements leading to more efficient computation, but a major disadvantage in comparison to SAFE and SBFEM is the necessity of a complete grid for a long bounded structure because unbounded structures cannot be modeled.

We will present SBFEM, because it combines many advantages: Unbounded domains can be modeled, and their dispersion curves can be computed. Additionally, bounded domains can be analyzed. The main part of the domain is modeled by the semi-analytical solution. This lowers the computational costs significantly. Furthermore, SBFEM allows the modeling of crack tips elegantly by describing the crack tip analytically [3], which is useful for Non-Destructive Testing. SBFEM can be coupled with finite element grids without additional workload. Higher order elements can be used to lower the computation cost.

The scope of this article is to point out for which problems SBFEM is advantageous and to give an introduction into the foundation and the possibilities of this method for the computation of ultrasonic waves. This does not include a careful guide how to compute the matrices. The reader is referred to the literature [4-8]. In this contribution, we mainly concentrate on the computation of dispersion curves with SBFEM but will finally present some examples for analyzing the wave propagation.

Overview of the Solution Process with SBFEM

The Scaled Boundary Finite Element Method can model elastic wave propagation in the frequency domain in a two- and three-dimensional space. For simplicity, we consider one fixed frequency:

$$\omega^2 \rho\, u = \nabla * \sigma(u), \quad (1)$$

where ω is the frequency, ρ the density, σ the stress and u the unknown displacement. This equation is valid inside the domain. The solution for a numerical approximation has always two main steps. The first step is to build a suitable description of the domain. The second step is to compute an approximation inside this domain.

In FEM for example, the first step is to build a grid as a description of the domain. The second step is to compute the approximation for the displacement at each node of the grid. In the following we will describe these two main steps for SBFEM.

Step 1: Description of the Domain in SBFEM

In this section, the description of the domain is given, at first of an unbounded and then of a bounded domain.

In SBFEM the description has two parts: i. a grid and ii. points which are scaled and/or rotated to the grid.

Dispersion curves are computed for unbounded domains. For these domains, either a one-dimensional grid – a line with nodes – or a single two-dimensional grid is used. The computational cost depends mostly on the number of nodes used in the grid.

Common examples for these domains are infinite plates, with one line in y-direction and points which are scaled in the x- and the z-direction (Figure 1 - left) [4]. Another example is an infinite hollow cylinder (Figure 1 - right) [5], with one line and points which are scaled in the z-direction and rotated in ϕ-direction. The third example is a infinite domain with an arbitrary cross section (Figure 2) [6], with a grid in the xy-plane and points scaled in the z-direction. In all figures 1-7 the scaling is marked by the dotted lines.

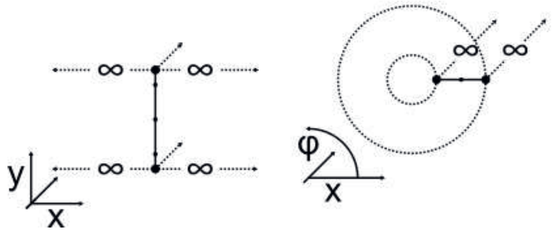

Fig. 1. Semi-infinite plate and hollow cylinder

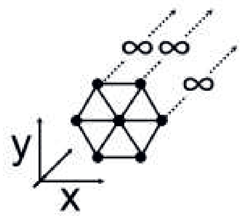

Fig. 2. Semi-infinite domain with arbitrary cross section

To model wave propagation in arbitrary domains, the domain sometimes must be decomposed in several sub-domains. There are three kinds of sub-domains with their own requirements.

The frist sub-domain consists out of a straight prismatic shape with two equal grids at each side (Figure 3). This is related to the infinite domain in Figure 2.

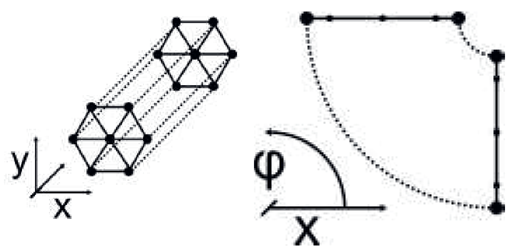

Fig. 3. Bounded straight and curved sub-domain

The second sub-domain is described by two equal grids which are linked by an arc of constant curvature [7]. This is shown in Figure 3 for the two-dimensional case. This sub-domain is related to the infinite hollow cylinder.

Figure 4 shows a curved sub-domain which is coupled with a straight sub-domain in two-dimensional space. The length of a sub-domain between the two cross sections does not affect the computational cost of the approximation. This is the reason, why SBFEM is especially suited for domains with large straight parts or large parts with a constant curvature. These domains are very common in the investigation with guided waves. In comparison, the more complicated finite element grid for the same situation is depicted in Figure 5.

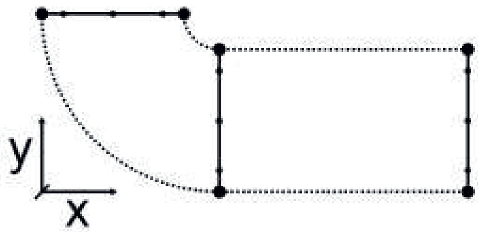

Fig. 4. SBFEM-grid

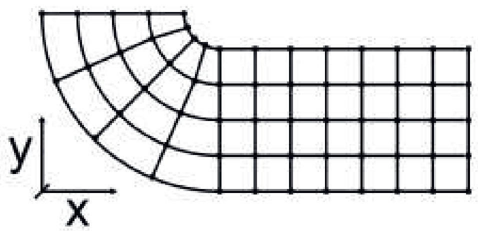

Fig. 5. FEM-grid

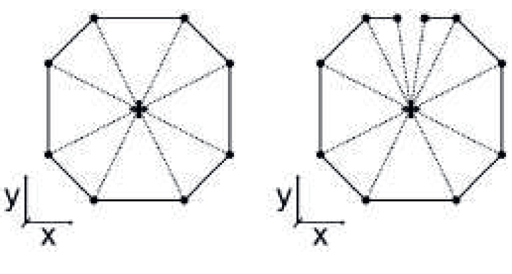

Fig. 6. Bounded star sub-domain

The last type of sub-domains must satisfy the so-called star property. This property postulates that there is one point - called scaling center - which is connected via straight lines to all boundary points [8]. In Figure 6 we can see an SBFEM-grid for an arbitrary shape in two dimensions. The right picture is a special case because the grid is not closed – this can be used to model cracks [3].

Another possibility to divide the domain is that the SBFE-grid can be coupled with an FE-grid. This is possible if both grids coincide. Figure 7 summarizes all coupling possibilities.

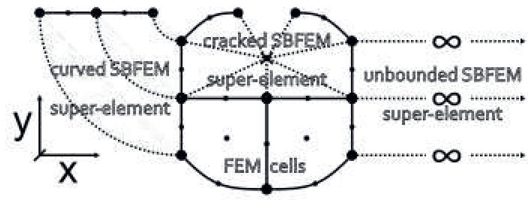

Fig. 7. Summary coupling possibilities

Step 2: Approximation of the displacement

In SBFEM the approximation of the displacement has also two parts: one finite element part depending on the grid and one analytical part. For simplicity, we consider only one scaling direction as shown in Figure 2.

If the z-direction is infinite, the solution of Equation (1) takes the form

$$\mathrm{u}(x,y,z,\mathrm{t}) = \underbrace{\tilde{u}(x,y)}_{\text{FE-part}} * \underbrace{e^{i(\omega t-kz)}}_{\text{analytical part}} . \quad (2)$$

There are only unique pairs (ω, k) of the frequency ω and the wavenumber k possible. The displacement for one pair is called a mode. Modes build the foundation of each test with guided waves, because every wave field that travels a long distance can be decomposed into modes. The aim is to excite only a few modes and compute the interaction of these modes with flaws. If a mode is reflected by a flaw the group velocity is needed to compute the position of this flaw.

For a fixed frequency, the SBFEM solution process leads to an eigenvalue problem

$$-Z\psi = \lambda\psi, \quad (3)$$

where λ is the eigenvector, ψ is the eigenvalue, and Z is a matrix depending on the grid.

As in FEM, the displacement is given for the nodes of the grid. Every other point of the domain can be computed by the FEM-interpolation or by Equation (2). The eigenvector $\psi = (\hat{u} \quad \hat{q})^T$ contains values of the displacement in the upper half of the vector. In the lower half are the force values to excite this mode for each node. The eigenvalue is directly linked with the wavenumber by

$$\lambda = ik. \quad (4)$$

The phase velocity is given by

$$v_p = \omega/k. \quad (5)$$

Only if λ is purely imaginary the eigenvector defines a propagating mode. The details for the derivation of the matrix Z for infinite plates can be found in [2], for infinite rods in [5,7], and for domains with an arbitrary cross section in [6].

Let $\psi = (\hat{u} \quad \hat{q})^T$ be a special eigenvector with a purely imaginary eigenvalue, then the group velocity can be computed as [10]

$$v_g = \frac{\hat{u}^* \hat{q} - \hat{q}^* \hat{u}}{2\omega\, \hat{u}^* M_0 \hat{u}}. \qquad (6)$$

Here $(\cdot)^*$ is the conjugate transpose operator and M_0 is a matrix which is very similar to the mass matrix in classic FEM.

Example 1

This example demonstrates the possibility to compute dispersion curves even for a more involved case with different layers.

In this example, we consider an IM7/8552 carbon-epoxy composite with a layup $[0\ 90\ 0\ 90]s$. This composite is used for example in the aerospace industry.

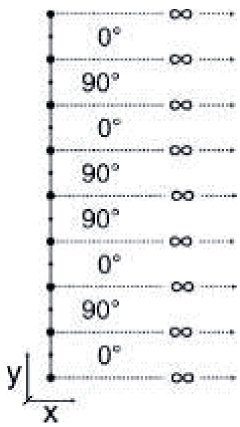

Fig. 7 Layup $[0\ 90\ 0\ 90]s$

The layup is summarized in Figure 7. It is assumed that every ply is transversally isotropic [9]. Each ply is 0,8 mm thick and the material properties are summarized in Table 1. The different layers are modeled by rotation of the transversally isotropic elasticity matrix. This example was computed with a plain-strain assumption to reduce the three-dimensional problem to two dimensions. The time of computation with an implementation in MATLAB and a modern computer for one frequency takes less than a second. Exact performance tests can be found in [3].

The dispersion curves are summarized in the Figures 8-10. The wavenumber in Figure 8 is computed by Equation (4) and plotted if the associated eigenvalue is purely imaginary. For each purely real wavenumber, the phase velocity is computed with Equation (5) and the group velocity is computed with Equation (6).

Tab. 1: Material properties

	IM7/8552	
ρ	1570 .00	kg/m^3
E_1	171.40	GPa
E_2	9.08	GPa
E_3	9.08	GPa
G_{12}	5.29	GPa
G_{13}	5.29	GPa
G_{23}	2.80	GPa
ν_{12}	0.32	
ν_{13}	0.32	
ν_{23}	0.50	

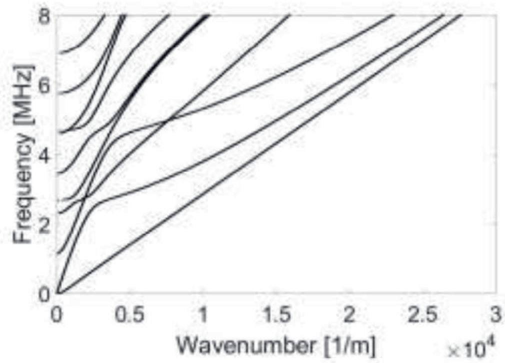

Fig. 8. Wavenumber vs Frequency in IM7/8552

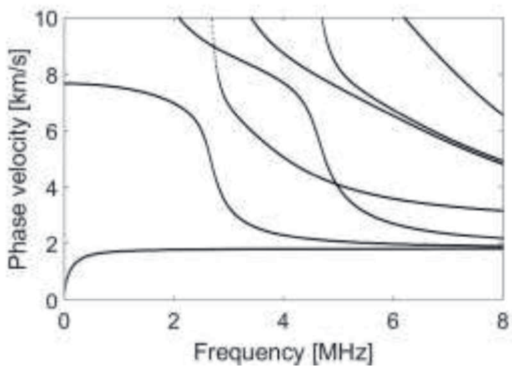

Fig. 9. Frequency vs Phase Velocity in IM7/8552

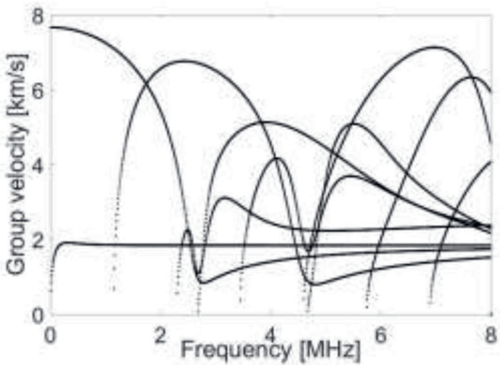

Fig. 10. Frequency vs Group Velocity in IM7/8552

Example 2

For the second example, we consider a waveguide with a cross-section pictured in Figure 11. We look only at one mode at a frequency of 3.14 MHz and a phase velocity of 3.25 km/s. The material is isotropic steal. The material parameters are summarized in Table 2.

The displacement of this mode is computed with Equation (2) and pictured in Figures 12-14.

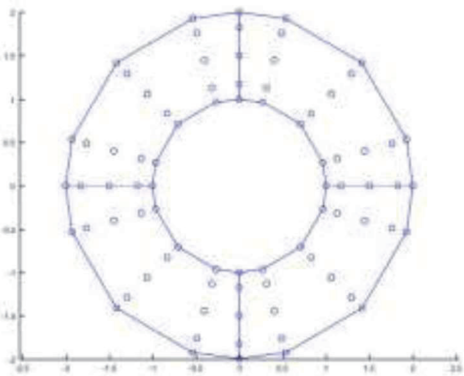

Fig. 11. Cross section grid for example 2

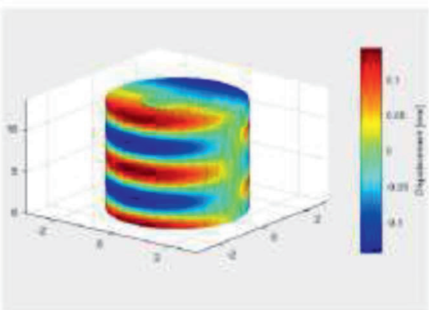

Fig. 12. Displacement in x-direction

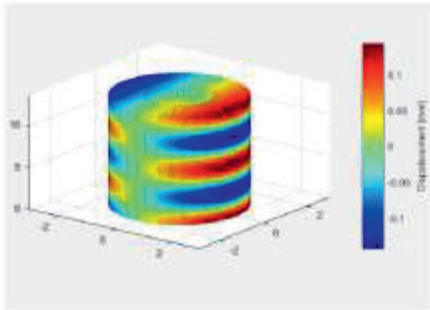

Fig. 13. Displacement in y-direction

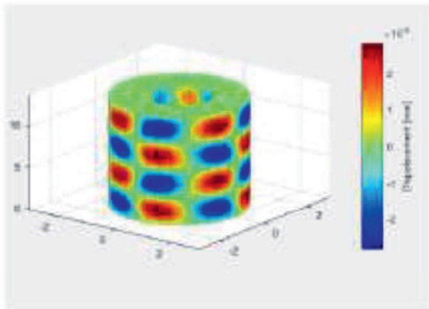

Fig. 14. Displacement in z-direction

Tab. 2: Material properties

	steal	
ρ	7.800	kg/m^3
G	82.64	GPa
ν	0.28	

The simulation of wave propagation

For simplicity, we consider only excitations at the grid. For straight and curved sub-domains the eigenvectors of Z are used to derive a stiffness-matrix $K(\omega)$ and the displacement u is derived by solving the linear matrix equation

$$K(\omega)\, u = F, \quad (7)$$

where F is associated with the external boundary forces on the nodes.

Detailed information about the derivation of the stiffness-matrix can be found for plates in [2] and for rods in [5,7]. For the third kind of sub-domain, the derivation is much more involved, it can be found in [8].

Example 3

This Example shows that SBFEM can compute interesting cases with only a few sub-domains. In this example, an ultrasonic wave travels in a bended pipe with a decomposition like in Figure 4. The displacement is computed with Equation (7).

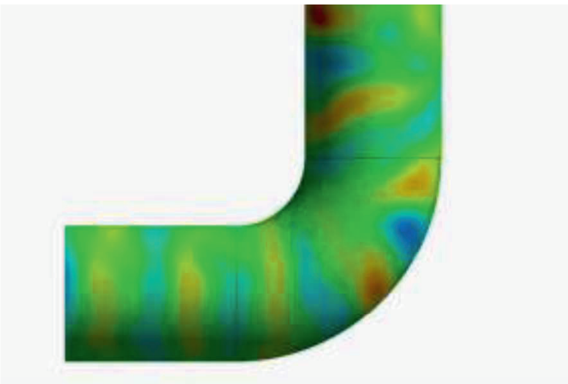

Fig. 15. Displacement in a bended pipe

Conclusion

The current state of research for the Scaled Boundary Finite Element Method for the simulation of guided wave propagation was summarized.

The Scaled Boundary Finite Element Method is a fast and suitable method to compute dispersion curves. Even layered structures can be computed.

A common structure which is tested with guided waves has large straight parts. These structures can be decomposed in only few sub-domains. It was shown that this decomposition leads to very efficient computation with SBFEM.

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