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Unambiguous Wide-Range Optical Voltage Sensor with Dual Operating Wavelengths

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Abstract:
We extend the unambiguous measurement range of an electro-optic voltage sensor from 75 kV to 100s of kilovolts (spanning multiple 2π phase shift ranges), by combining phase shift principal value measurements at two different wavelengths. Compared with previous two-wavelength techniques, this new approach completely eliminates all remaining measurement ambiguities.

Key words: phase shift measurement, two-wavelength measurement, electro-optic voltage sensor, period disambiguation.

Introduction

There has been great interest in the electric power industry in developing new voltage sensors based on optical measurement principles. Compared with conventional technologies, the optical sensors are compact, galvanically insulated, insusceptible to electro-magnetic interference, and cost-effective. For emerging HVDC applications (e.g. offshore wind parks and long-distance power transmission), optical voltage sensors are even more attractive, as conventional capacitive and inductive transformers cannot measure DC voltages.

Optical voltage sensors rely on the interference between two orthogonal polarization modes. The applied voltage induces a relative phase shift, which is measured by photodetectors. For an electro-optic voltage sensor, bismuth germanate ($Bi_4Ge_3O_{12}$, or BGO) of the $\bar{4}3m$ symmetry class is a preferred sensing medium. Particularly advantageous is the longitudinal configuration, in which the [001] crystal axis is oriented along the optical path so that the measured phase shift is proportional to the path integral of the electric field $\int \boldsymbol{E} \cdot d\boldsymbol{s}$, i.e., the applied voltage between the crystal end faces. The field integration produces an accurate measurement of the voltage, irrespective of space charge and local field distributions.

Typically, an electro-optic voltage sensor uses the sensing crystal together with some polarizers and waveplates [1]. The output of such a polarimetric sensor is a sinusoidal function of the differential phase shift between two orthogonal polarizations. Phase shifts of ϕ and $\phi + 2n\pi$ (n being an integer number) produce the same polarimetric output, and hence cannot be distinguished from one another. (For an individual polarimetric signal, there is an additional sign ambiguity within a single period, i.e., $\cos\phi = \cos(-\phi)$, which can be removed by adding a quadrature signal $\sin\phi$.)

The periodwise ambiguity is an inherent problem for all interferometric measurements. For relative measurements, the measurement range can be extended by fringe-counting or similar history-tracking techniques. In AC voltage measurements, one can thus extend the measurement range to many times the π-voltage (75 kV at 1310 nm for BGO) by combining quadrature polarimetric signals and using zero-crossing counting [2, 3], facilitated by the fact that the AC voltage continuously oscillates about zero.

On the other hand, for absolute measurements without history information, the periodwise ambiguity places a fundamental limit to the achievable unambiguous measurement range. Such is the case for the measurement of the DC voltage, which lacks an oscillating waveform and thus a zero voltage reference. Furthermore, the lack of zero reference also makes it difficult to distinguish voltage drifts from effects such as changing optical loss, stress-induced birefringence, etc. Kurosawa and Yoshida attempted to address the drift problem by chopping the applied voltage [4], but such solutions are not amicable to HV applications. Therefore, a wide-range (100s of kilovolts) optical DC voltage sensor calls for a different solution.

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Deficiencies of previous dual-wavelength period disambiguation techniques

One way to extend the unambiguous range of a phase shift measurement is to combine it with another unambiguous (but coarser) measurement. For some applications, one can simply use two types of sensing medium for high- and low-resolution measurements, respectively [5]. This is however not an option for voltage sensing, due to the scarcity of suitable sensing crystals of different sensitivities.

It has been suggested [6, 7] to solve the periodwise ambiguity by combining measurements at two wavelengths of different voltage sensitivities. Although the measurement at either wavelength is still periodic, the dual-wavelength pair as a whole generally does not have a periodic dependence on the voltage, and hence can be used to unambiguously allocate the combined sensor's output in a range much larger than either sensor's unambiguous range. The dual-wavelength sensor outputs are then

$$y_1(V) = \cos\phi_1 = \cos(q_1 V + \delta_1)$$

and

$$y_2(V) = \cos\phi_2 = \cos(q_2 V + \delta_2)$$

where $q_1 \neq q_2$ are the voltage sensitivities of the electro-optic phase shifts $\phi_{1,2}$ at the two wavelengths $\lambda_{1,2}$, and $\delta_{1,2}$ are the phase offsets (set to 0 in later simulations). For the BGO voltage sensor, $q = 2\pi n^3 r_{41}/\lambda$, where n is the refractive index, and r_{41} is the electro-optic coefficient. For simplicity and without loss of generality, here the amplitudes of the outputs y_1 and y_2 are set to 1, and any potential offsets to 0.

While the dual-wavelength period disambiguation seems straightforward at the first glance, an important caveat was disregarded in previous publications. It becomes evident when the pair $Y(V) = [y_1(V), y_2(V)]$ is plotted as the voltage changes, resulting in a Lissajous figure (see Fig. 1a). The shape of a Lissajous figure is characteristic of the ratio q_1/q_2 as well as the phase offset difference $\delta_1 - \delta_2$. Therefore, it is widely used to visually represent the relationship between harmonic signals.

The Lissajous figure maps a 1D variable to a point along the trace $Y(V)$ in the 2D y_1-y_2 plane. If the ratio q_1/q_2 is rational, i.e., $q_1/q_2 = N_1/N_2$, where N_1 and N_2 are integers, the trace is a closed curve, and the 2D period

$$d = 2\pi N_1/q_1 = 2\pi N_2/q_2$$

is increased N_i-fold from the single-measurement period $2\pi/q_i$. If q_1/q_2 is irrational, the trace is not closed, meaning the two-wavelength measurement $Y(V)$ is aperiodic. Therefore, one can use the dual-wavelength method to significantly increase the unambiguous range of the polarimetric measurement.

It is also obvious from the Lissajous figure that the trace makes many crossings upon itself as the voltage progresses. At any crossing point in the figure, there exist two possible voltage values that produce the same sensor output $Y(V)$. Therefore, one crossing point corresponds to a pair of two possible voltage values that cannot be mutually distinguished in the measurement.

One solution to this problem has been proposed by adding a third measurement at a different wavelength [7]. The trace in the 3D space $Y(V) = [y_1(V), y_2(V), y_3(V)]$, known as a Lissajous knot, generally does not make crossings upon itself as the voltage progresses (with the exception of some isolated degenerate cases). Therefore, no ambiguity generally exists. However, operating light sources, detectors and other optical components at three or more wavelengths increases the complexity and cost, and reduces the reliability of the entire sensor system. Furthermore, mapping a 3D (or higher-dimensional) measurement to a single measurand value also involves more complicated signal processing. Therefore, it is not a preferred approach to solve the ambiguity problem.

The properties of this ambiguity can be better studied by performing an arccos transformation on the Lissajous figure.

$$Z = [z_1, z_2] = [\arccos y_1, \arccos y_2]$$

where arccos returns the principal value of the inverse cosine function, defined in the range [0, π]. Note that since the arccos transformation is a bijective (invertible one-to-one) mapping from [-1, 1] to [0, π], *Y* and *Z* are equivalent representations of the same data. Making use of the equality $\arccos(\cos\phi) = |\mathrm{pv}\,\phi|$ results in:

$$Z = [z_1, z_2] = [|\mathrm{pv}\,(q_1 V + \delta_1)|, |\mathrm{pv}\,(q_2 V + \delta_2)|]$$

where the function pv yields the principal value of a phase angle defined in the range (-π, π].

Because z_1 and z_2 are each segmented linear functions of *V*, the *Z* trace, plotted in Fig. 1b, consists of a series of straight lines, and the voltage V is uniformly distributed along these lines. The entire *Z* trace can be viewed as the propagation of a straight ray as it is being continually "reflected" at the four boundaries of a square box, defined by $z_{1,2} = 0, \pi$. Therefore, half of the segments have a positive slope q_2/q_1, and the other half a negative one $-q_2/q_1$.

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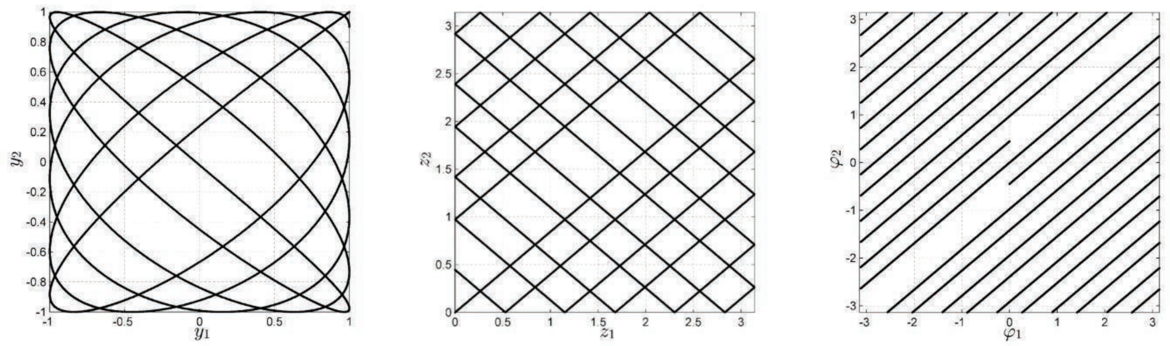

Fig. 1. From left to right: (a) Y trace (Lissajous figure), (b) Z trace, (c) Φ trace. For these plots, a double-pass BGO voltage sensor is modeled at 1310 and 1550 nm with $\delta_1 = \delta_2 = 0$ within [-450kV, 450kV]. Correspondingly in this example, q_1 = 0.0838 rad/kV, and q_1 = 0.0708 rad/kV.

Ambiguity is created by the crossing between a positive-slope segment and a negative-slope segment. All the positive-slope segments are parallel to and evenly spaced between each other, as is true also for the negative-slope segments. Consequently, the crossing points are also evenly spaced along the segments. Because the voltage *V* is uniformly distributed along the straight lines, this also means that the ambiguity points are quasi-uniformly distributed in the entire measurand range.

Some further mathematical analysis will show that the total number of crossing points is roughly $M = N(N-1)/2$, where $N = \lfloor (q_1 + q_2)L/2\pi \rfloor$, and L is the size of the measurement range. Obviously, since *N* is linear with *L*, the total number of ambiguous measurand values 2*M* increases quasi-quadratically with the measurand range size.

For a BGO voltage sensor, the π-voltage is about 75 kV at 1310 nm, and 88.7 kV at 1550 nm. If the double-pass polarimetric outputs at these two wavelengths are used, in the voltage range [0, 450 kV], the total number of ambiguous voltage values is 100, meaning on average one ambiguous voltage value occurs every 4.5 kV.

The large number of ambiguity points in a wide-range dual-wavelength polarimetric sensor presents a challenge for signal processing. One might argue that, because ambiguity always occurs between two segments of opposite slopes, it is possible to use the time derivative of the measured phase shift to determine on which segment the ambiguous measurement should fall. This approach, however, requires tracking the measurement history, and assumes that the measurand waveform has a meaningfully large local derivative when the ambiguity point occurs. For voltage measurements, this approach might work for AC voltages, but would not work reliably for DC voltages. Furthermore, signal processing is especially problematic for the measurement of fast transient waveforms, in which the sample-to-sample change of the measurand can be larger than the spacing between adjacent ambiguous voltage values.

Complete period disambiguation with 2π-range dual-wavelength detection

Suppose, by a suitable method, the principal values of the phase shifts are determined unambiguously within a 2π range, i.e.,

$$\Phi = [\varphi_1, \varphi_2] = [\mathrm{pv}(q_1 V + \delta_1), \mathrm{pv}(q_2 V + \delta_2)]$$

A plot of Φ in the φ_1-φ_2 plane as the voltage *V* progresses is shown in Fig. 1c. This Φ trace consists of straight line segments, now all of the same slope q_2/q_1.

The Φ trace can also be envisioned as the propagation of a ray inside a square box enclosed by boundaries $\varphi_{1,2} = \pm\pi$. When the ray reaches a boundary, it reemerges from the same position on the opposite side, and continues with the same slope. Therefore, all segments in the trace are parallel to each other, and are quasi-equally spaced, with no crossing or associated ambiguity points.

Any method that unambiguously determines the phase shift principal value (at each of the wavelengths) can be used for the implementation of this approach. For example, in a quadrature detection scheme, one measures two quadrature signals

$$I_1 = I_0(1 + A\cos\phi)/2$$

$$I_2 = I_0(1 + A\sin\phi)/2$$

If the total light power I_0 is also measured, a complex variable S can be calculated as

$$S = \left(\frac{2I_1}{I_0} - 1\right) + i\left(\frac{2I_2}{I_0} - 1\right) = Ae^{i\phi}$$

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Therefore, the phase shift principal value in the 2π range is

$$\varphi = \mathrm{pv}\,\phi = \arg S$$

Other phase measurement techniques, such as the non-reciprocal phase modulation scheme widely used in fiber-optic gyroscopes [8] and current sensors [9, 10], may also be used for the dual-wavelength period disambiguation, as long as the phase at each wavelength can be individually determined in a 2π range.

A wide-range optical voltage sensor can then be designed using the 2π-range dual-wavelength period disambiguation method and polarimetric quadrature detection, as shown in Fig. 2.

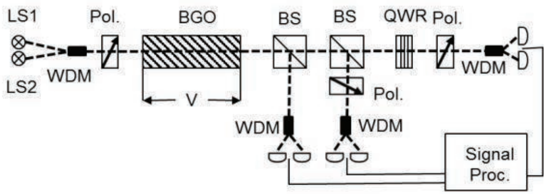

Fig. 2. Schematic of a wide-range unambiguous BGO voltage sensor. LS is light source, WDM wavelength-division multiplexer, Pol. polarizer, BS beamsplitter, QWR quarter-wave retarder, D detector.

There are several different approaches with regard to the signal processing procedure of converting the measured dual-wavelength outputs to an unambiguous voltage value.

One approach is the following: After measuring the phase shift principal values $[\varphi_1, \varphi_2]$ at the two wavelengths, one generates two lists of possible full values of the phase shifts $\left[\phi_m^{(1)}, \phi_n^{(2)}\right] = [\varphi_1 + 2m\pi, \varphi_2 + 2n\pi]$ (*m* and *n* are integers) in the measurement range. Then, two lists of possible voltage values are calculated from these phase shifts

$$\left[V_m^{(1)}, V_n^{(2)}\right] = \left[\left(\phi_m^{(1)} - \delta_1\right)/q_1, \left(\phi_n^{(2)} - \delta_2\right)/q_2\right]$$

Next, the two lists can be compared to identify a pair of $\left[V_m^{(1)}, V_n^{(2)}\right]$ with the smallest difference $\left|V_m^{(1)} - V_n^{(2)}\right|$. Finally, the output can be set as the average of both voltages $\left(V_m^{(1)} + V_n^{(2)}\right)/2$.

A second method is the following: Each segment in the Φ trace can be uniquely labeled with a pair of indices $[m, n]$ by the relationship

$$\begin{aligned}[\phi_1, \phi_2] &= [q_1 V + \delta_1, q_2 V + \delta_2] \\ &= [\varphi_1 + 2m\pi, \varphi_2 + 2n\pi]\end{aligned}$$

Therefore, one can define

$$\begin{aligned}\Delta_{mn} &= q_1\varphi_2 - q_2\varphi_1 \\ &= 2\pi(mq_2 - nq_1) \\ &\quad + (q_1\delta_2 - q_2\delta_1)\end{aligned}$$

Mathematically, $\Delta_{mn}/\sqrt{q_1^2 + q_2^2}$ is the signed perpendicular distance from the origin to the segment with the index pair $[m, n]$, with its sign indicating on which side of the origin the segment lies. Therefore, in the entire unambiguous measurement range, each index pair $[m, n]$ corresponds to a unique Δ_{mn} and vice versa. This mapping can be pre-calculated and saved in a 1D tabular form. An example of the one-to-one correspondence between Δ_{mn} and $[m, n]$ is shown in Fig. 3.

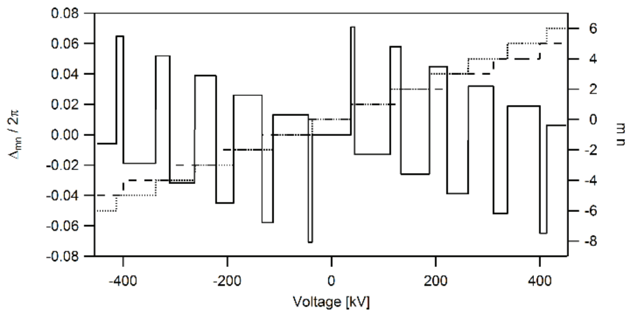

Fig. 3. Segment judgment diagram modeled for a double-pass BGO voltage sensor operating at 1310 nm and 1550 nm, with $\delta_1 = \delta_2 = 0$ in a measurement range [-450 kV, 450 kV]. The solid line is the calculated Δ_{mn}, the dotted line the q_1 segment number m, and the dashed line the q_2 segment number n. Note that there is a one-to-one correspondence between the index pair $[m,n]$ and the Δ_{mn} value.

Hence, from the measured phase shift principal values $[\varphi_1, \varphi_2]$, one can calculate Δ_{mn} and look up the corresponding indices $[m, n]$ from the pre-calculated table. Finally, the full values of the phase shift and the corresponding measurand value can be calculated.

The first method involves a search in a 2D space consisting of two dynamic lists, whereas the second method involves only a 1D lookup in a static list. Therefore, in terms of computational complexity, the second signal processing method is much more efficient.

Guideline for optimal wavelength selection

We present here a design guideline for the selection of optimal wavelengths for a dual-wavelength interferometric sensor.

We start with one given wavelength λ_1 and the corresponding q_1. For simplicity, it is assumed that $q_1 = 2\pi N_1/L$. Therefore, the measurement at λ_1 results in N_1 line segment end points on the left (or right) boundary of the 2D phase space (-π/2, π/2] × (-π/2, π/2].

It is generally advantageous to separate all line segments in Fig. 1c as far away as possible between each other, in order to minimize the influence of measurement noise. This can be best realized if the segments uniformly fill the 2D phase space from corner to corner.

To satisfy this uniform filling condition, the end points on the vertical phase-space boundary should be evenly distributed, i.e., the separation between two adjacent end points should be $2\pi/N_1$. This can be achieved by making the measurement at λ_2 fill a 2π range, i.e., $q_2 = 2\pi/L$. And even more generally, the condition is satisfied for all $q_2 = 2\pi N_2/L$ as long as N_1 and N_2 are coprime, i.e., their greatest common divisor is 1. This leads to the following criterion:

$$\Delta q = 2\pi\Delta N/L$$

where $\Delta q = |q_1 - q_2|$ and $\Delta N = |N_1 - N_2|$.

The formula above provides a list of candidate wavelengths that satisfy the uniform-filling condition. For a BGO voltage sensor with λ_1 = 1310 nm and a measurement range of [-450 kV, 450 kV], the list of the optimal second wavelengths is $\lambda_2 \approx \lambda_1 (1 \pm 0.0781\Delta N) \in$ {1215 nm, 1421 nm, 1133 nm, 1553 nm, ⋯}. Since 1550 nm is a standard telecom wavelength at which many commercial components are available, it would be a particularly ideal choice for use in a dual-wavelength BGO voltage sensor.

Summary

We show that the unambiguous measurement range of an interferometric sensor can be extended to many phase periods, by combining two 2π-range phase shift measurements at distinct wavelengths. Compared with previous dual-wavelength techniques, our approach eliminates all measurement ambiguities, and is particularly suitable for an optical DC voltage sensor up to several 100s of kilovolts. Based on the mathematical properties of the signal, we have also developed a particularly efficient signal processing algorithm, which performs a fast 1D table lookup to convert the dual-wavelength sensor output to the corresponding voltage value. A guideline for choosing the optimal sensor wavelengths for a given measurement range is also presented.

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