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Using the Kibble Principle for One-Step Traceability for Mass, Force, and Torque

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Summary:

Many technologies contributed to the 2019 revision of the international system of units. Notable is a mechanical apparatus, the Kibble balance. It allows the precise comparison of electrical power to mechanical power with relative uncertainties close to 1 part in 10^8. Here, we show the basic principle of this device and discuss several exciting future applications.

Keywords: Kibble balance, revised SI, force measurements, torque measurements

Introduction

In 1975, Bryan Kibble, a metrologist at the National Physical Laboratory in the UK, had an insight [1] that would eventually lead to the revision of the international system of units (SI) in 2019. As is the case with many eureka moments, in hindsight, they are obvious, but they solve a long-standing problem. In this particular case, Kibble's ideas allowed us to obtain precisely the force of a current-carrying wire in a magnetic field, an almost century-old struggle at the time. And precise it is. The world's best measurements utilizing Kibble's idea have relative uncertainties of 1 part in 10^8. Clearly, such a powerful insight must be treasured and understood. An explanation is attempted below.

Kibble's idea in a nutshell

The energy of a coil in a magnetic flux density, B, is given by the number of turns, N, times the current, I, encircling the area of the coil, A, or

$$E = NIBA. \qquad (1)$$

Note, the product BA is the flux through the coil opening, abbreviated as $\phi = BA$. From eq. (1), the forces and torques on the coil can be obtained by the partial derivatives in the corresponding direction. So, for example, the force in the z direction is given by

$$F_z = -\frac{\partial E}{\partial z} = -NI\,\frac{\partial \phi}{\partial z}. \qquad (2)$$

Even in 1975, this was nothing new. People have tried to measure forces using eq. (2), but the problem is that the product BA is difficult to know precisely. Absolute measurements of B are cumbersome and what exactly is the open area of a coil, A? Even if the coil is made from a single layer how much of the wire diameter must be considered to calculate A?

Kibble noticed that the flux through the coil appears in another equation, Faraday's law of induction,

$$U = -N\,\frac{d\phi}{dt}. \qquad (3)$$

Here U is the electromotive force (EMF) that appears at the open-ended leads of a coil as the flux through the coil varies with time. Now assume, the coil is moved, by some undescribed mechanism, through a magnetic field that is constant in time but not in space in a purely vertical trajectory, the time derivative can be replaced in the following way,

$$U = -N\,\frac{\partial \phi}{\partial z}\frac{d\,z}{d\,t} = -N\,\frac{\partial \phi}{\partial z}\,v_z. \qquad (3)$$

By measuring the induced EMF and the vertical velocity, v_z, the derivative of the flux can be obtained. What makes the idea powerful is that both quantities can be measured easily and precisely.

Eliminating the flux integral in eq. (2), yields

$$F_z = \frac{I\,U}{v_z}. \qquad (4)$$

In the section above, we worked an example in the vertical direction, as is the case in the Kibble balance. The theory works, of course, in all directions and even for rotational motion. The torque, N_x, about an axis *x* can be measured using

$$N_x = \frac{I\,U}{\omega_x}. \qquad (4)$$

In eq. (4), ω_x is the angular velocity about the same axis while *U* is measured.

Work at the National Institute of Standards and Technology (NIST)

In the 2010s, a Kibble balance for a nominal mass of 1 kg was built with the purpose to aid the international effort to revise the SI and to become the primary mass standard of the United States after said revision [2]. This Kibble balance produces measurements with competitive uncertainties for masses ranging from 50 g to 2 kg [3].

After the 2019 revision of the SI, the big Kibble balance remains operative, and research has expanded to table-top-sized Kibble balances. Figure 1 shows one such balance, KIBB-g2. This balance has a load capacity ranging from 500 mg to 20 g. The goal is to measure these masses with relative uncertainties of 1×10^{-5}.

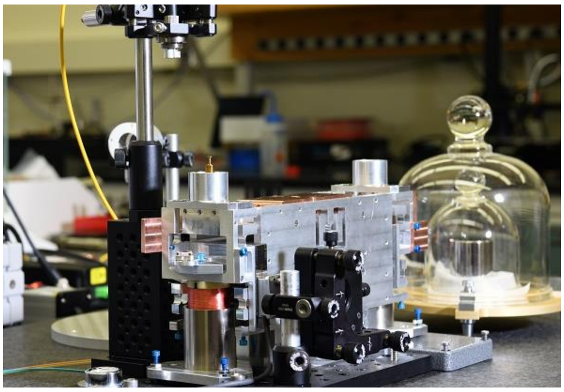

Fig. 1. Photograph of KIBB-g2. A Kibble balance that fits on a table and can be used to measure gram-level masses. Photo credit: Curt Suplee/NIST.

Furthermore, we have started a new project aiming to build a device that can calibrate torque. Our first model can calibrate torques of order 18 mN·m with relative uncertainties of order 10^{-3}.

A xylophone of devices

The big Kibble balance at NIST has shown to be able to measure masses within a factor of 40. If one is willing to incur larger uncertainties, that range can be stretched to about 100 or two decades. Hence, to cover the range from 1 mg to 1 kg, about three Kibble balances are necessary. For the smaller masses, below 30 mg, the magnetic force on a current-carrying coil is too strong and it is better to use the force between two charged capacitor plates. To cover the desired range mentioned above one would need one electrostatic balance and three Kibble balances.

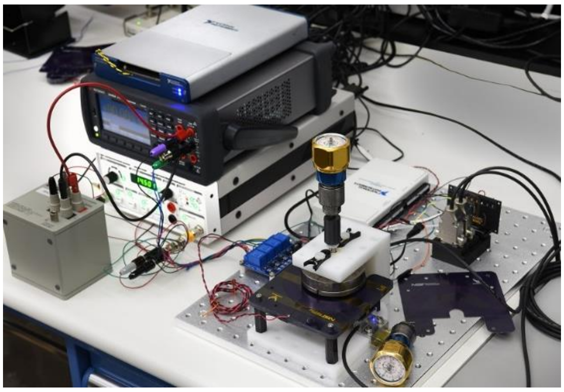

Fig. 1. The electronic NIST torque realizer. A device that calibrates torque watches at torque levels of 1 Nm with relative uncertainties of 10^{-3}. Photo credit: Curt Suplee/NIST.

For the torque project, we have shown the performance on a device that can measure torques ranging from order 1 mN·m to 18 mN·m. The next step is to build devices for larger torque ranges.

Conclusion

The reciprocity in Maxwell's equations that Kibble saw proved to be a powerful principle for high-precision metrology. However, these principles could be useful in mass-produced devices. We have shown table-top instruments that can calibrate mass, force, and torque using Kibble's principle.

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