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A Time Domain Measurement Technique for the Frequency Response of Voltage Controlled Capacitors

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Summary: This paper proposes a measurement technique to analyse the frequency response to changes in the control voltage of variable capacitors (varactors) with capacitance controlled by an external voltage. Examples are based on micro-electro-mechanical systems (MEMS) or ceramic components. The measurement principle is based on an RC circuit to which a continuous wave signal at the corner frequency of the resulting filter structure is applied. Equations are derived to determine the capacitance values as well as the frequency response of the varactor. Finally, a test setup is presented that verifies the results using a commercially available ceramic varactor.

Keywords: Varactor, micro-electro-mechanical systems (MEMS) Varactor, MEMS Capacitor.

Introduction

The measurement of capacitance is a long used and well known measurement application. There are many different methods to measure a static capacitance, as for example time constant measurements and impedance measurements [1] as well as resonance based measurements [2]. With the increased use of time variable capacitances (varactors), as for example used to tune resonant circuits and other electronic circuits, the analysis of the frequency response and response time of varactors becomes more important.

For two-port varactors using different ports for control of the capacitance and signal, different approaches have been made to measure the reaction time and frequency response to changes in the control voltage. For MEMS-capacitors the dynamic capacitance can be easily determined during manufacturing by monitoring the mechanical motion of the capacitor plates [3]. However, this is not possible for packaged varactors and embedded topologies for which moving elements are not accessible. Another approach is to measure displacement current that is created by the movement of a charged capacitor [4]. The displacement current however can be very small for low capacitances and voltages and then requires a very high measurement accuracy. A more robust but also more complex measurement setup is shown in [5], where the nonlinear properties of the continuously changing capacitance are used to create an intermediate frequency. The latter can be filtered or directly measured using a spectrum analyzer. This measurement principle however is very complicated to set up and requires lots of external components. In [6] another setup is shown that measures the frequency response of a system directly in time domain, however the signal generation is relatively complex as a frequency ramp is required.

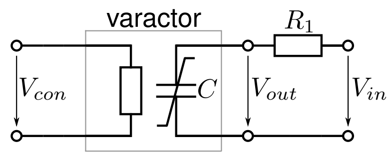

Fig. 1: Parameter definitions and measurement setup for frequency response measurement of a two-port varactor.

As an alternative, we propose a simple measurement setup that allows the measurement of capacitance change based on an RC-circuit and continuous wave (CW) waveform generation. RC elements are widely used for capacitance measurement [7][8]. Most principles use the phase shifting properties of RC element to accurately determine the capacitance value. This work will focus on amplitude measurement of the RC circuit near the corner frequency of the low pass filter model, as changes of this property are very easily observable over time using an oscilloscope. From measurements of the signal amplitude the frequency response of the varactor to changes in the control voltage can be determined.

Measurement Principle

The proposed measurement principle is based on a first order low pass filter, containing the varactor under test (DUT) and a known resistor R_1. Figure 1 shows the basic measurement setup and voltage definitions.

An AC voltage V_{in} is applied to the low pass filter by a signal source. The applied voltage is attenuated by the RC element, depending on the sizes of resistor and capacitor, and can be measured as V_{out} using an oscilloscope. The voltage

V_{con} controls the capacitance C as specified by the manufacturer of the varactor.

When modelling the varactor's response to an applied control voltage as a low pass behavior, a mechanical corner frequency ω_m can be defined. The frequency of the input voltage ω_{in} should be set to a value significantly larger than the mechanical corner frequency ω_m for two reasons: Firstly, the measurement frequency should be high enough to not have any effect on the varactor's behavior. Secondly, the change in amplitude of the measurement signal will indicate changes in the capacitance. Therefore, the dynamic of the measurement signal should be significantly higher than the highest possible dynamic of the varactor to improve the time resolution. For measurements up to the corner frequency the Nyquist theorem is a good approximation for the lowest usable signal frequency, so that

$$\omega_{in} > 2\omega_m. \tag{1}$$

R_1 is used to set the RC element's corner frequency to the input frequency ω_{in}. For the capacitance an initial capacitance value C_s from within the varactor's range is selected.

$$R_1 = \frac{1}{\omega_{in} C_s}. \tag{2}$$

The amplitude of the input signal $\hat{V}_{in}$ has a lower limit set by the measurement accuracy of the voltage detection of $\hat{V}_{out}$. As a voltage ratio is measured, the higher $\hat{V}_{in}$ the more accurate the measurement is. On the other hand, a high voltage across the signal port may interfere with the DUT's performance, e.g. causing self-actuation. Information provided by the manufacturer should be consulted to avoid unwanted effects or destruction of the DUT.

From the measured voltage V_{out}, with different control port signals applied, the relative capacitance change as well as the frequency response of the varactor can be derived. This will be discussed in section .

Derivation of Varactor Parameters

In this section the derivation of the capacitance based on the measured voltage V_{out} will be explained. This will be done for the setup shown in section and for a slightly different setup that allows adapting the tuning ratio. In section basic techniques to determine the varactor's frequency response will be discussed.

Capacitance

Defining a tuning factor of the capacitor a that relates the capacitance at an instance in time to the initial capacitance C_s linearly, so that

$$C = aC_s \quad \text{and} \quad \omega_{in} = \frac{1}{R_1 C_s} \tag{3}$$

the relation between measured output voltage amplitude $\hat{V}_{out}$ and the tuning factor a can be derived. For this, the ratio between $\hat{V}_{out}$ and $\hat{V}_{in}$

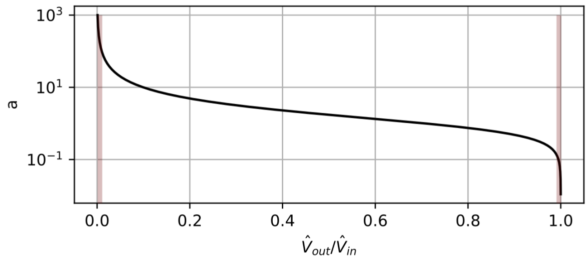

Fig. 2: Relation of tuning factor a to the ratio between measured voltage amplitude $\hat{V}_{out}$ and the input voltage amplitude $\hat{V}_{in}$.

using the definitions

$$V_{in} = \hat{V}_{in} e^{j\phi_{in}} \quad \text{and} \quad V_{out} = \hat{V}_{out} e^{j\phi_{out}} \tag{4}$$

is used. The relation between both signals is then described by

$$\hat{V}_{out} e^{j\phi_{out}} = \frac{\hat{V}_{in} e^{j\phi_{in}}}{1 + j\omega_{in} R_1 C}. \tag{5}$$

Solving equation 5 for a using the relations from equation 3 results in

$$\hat{V}_{out} = \frac{\hat{V}_{in}}{|1 + ja|} \quad \rightarrow \quad a = \sqrt{\left(\frac{\hat{V}_{in}}{\hat{V}_{out}}\right)^2 - 1}. \tag{6}$$

The relation is plotted in figure 2.

From Fig. 2 it can be seen that the measured amplitude ratio can be directly converted into the tuning factor a. However, the measurement resolution for both parameters needs to be very fine as the tuning ratio a increases or decreases. The highlighted areas in Fig. 2 show the values for a which are no longer distinguishable with a measurement resolution of

$$\Delta\hat{V} = 0.01 * (\hat{V}_{out}/\hat{V}_{in}). \tag{7}$$

which are approximately $a_{min} \approx 0.14$ and $a_{max} \approx 100$. The minimum tuning factor detectable with $\Delta\hat{V}$ is $\Delta a_{min} \approx 0.02 * C_s$.

In case the tuning ratio of the capacitor requires measuring values for a that are greater than the limits of the voltage measurement accuracy allows, the tuning ratio can be artificially decreased by adding a known capacitance C_d in parallel to the capacitance under test. This known capacitor C_d changes the tuning factor to

$$a' = a + \frac{C_d}{C_s} \tag{8}$$

and the resonance frequency to

$$\omega'_{in} = \frac{1}{R_1(C_s + C_d)}. \tag{9}$$

Using (5) and solving for a with

$$b = \frac{C_d}{C_s}. \tag{10}$$

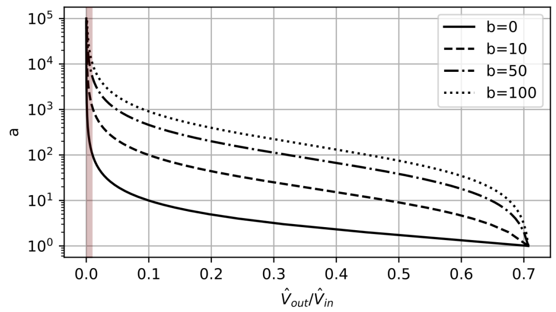

Fig. 3: Relation of tuning factor a to the ratio between measured voltage amplitude $\hat{V}_{out}$ and the input voltage amplitude $\hat{V}_{in}$ for different ratios b between the parallel capacitance C_d and the inital capacitance C_s.

results in

$$a = -\frac{b}{2} - \sqrt{\left(\frac{b}{2}\right)^2 + \left(\left(\frac{\hat{V}_{in}}{\hat{V}_{out}}\right)^2 - 1\right)(1 + b + b^2)} \tag{11}$$

from magnitude measurement.

Figure 3 shows the resulting relation between tuning factor and magnitude for different values of b. It can be seen that an increase in tuning factor a leads to a significantly smaller decrease in $\hat{V}_{out}/\hat{V}_{in}$ for higher values for b. The highlighted area again shows the values for a indistinguishable with a measurement resolution $\Delta\hat{V}$ as specified in (7). This shows that for higher values of a can be distinguished using the same measurement accuracy.

Frequency Response

The RC element and control voltage can be described as a two port device using V_{con} as input and the capacitance value

$$C = C_s \sqrt{\left(\frac{\hat{V}_{in}}{\hat{V}_{out}}\right)^2 - 1} \tag{12}$$

as output. This system can then be modelled using a transfer function. Depending on the device under test, a transfer function model can be created and the parameters of the device determined. As an example, this will be done for a first-order low pass filter model for which can be defined by a corner frequency ω_m. Two approaches to determine ω_m are shortly presented and will be verified in section .

Amplitude Measurement

The first approach is to measure the corner frequency directly. This is done by applying a sinusoidal signal with amplitude $\hat{V}_{con}$ and offset V_0 to the control port of the varactor. The steady state capacitances for the center

$$C_0 = C|_{V_{con}=V_0}, \tag{13}$$

minimum

$$C_{min} = C|_{V_{con}=V_0-\hat{V}_{con}}, \tag{14}$$

and maximum

$$C_{max} = C|_{V_{con}=V_0+\hat{V}_{con}} \tag{15}$$

values of the control voltage are determined. The frequency of V_{con} can then be altered until the minimum and maximum capacitances are determined to reach

$$C = \frac{C_{max} - C_0}{\sqrt{2}} + C_0|_{V_{con}=V_0+\hat{V}_{con}} \tag{16}$$

and

$$C = C_0 - \frac{C_0 - C_{min}}{\sqrt{2}}|_{V_{con}=V_0-\hat{V}_{con}}. \tag{17}$$

The frequency at which both conditions (16) and (17) are fulfilled is the corner frequency ω_m.

Step Response

The second approach uses the step response of the system to measure the time constant τ. To do so, a voltage step is applied to the control port of the varactor while the capacitance is at an initial value of C_0. Measuring the time between the voltage step and the moment at which the capacitance C reaches $C = C_0/e$, gives the time constant τ. From this the corner frequency

$$f_m = \frac{\omega_m}{2\,\pi} = \frac{1}{2\,\pi\,\tau} \tag{18}$$

can be calculated.

Validation Measurements

To verify the proposed methodology an example measurement was conducted using the ceramic variable capacitor *LXRW19V201-058* by *Murata Manufacturing Co.*. The measurement is set up as shown in Fig. 1 with the control voltage V_{con} and input signal V_{in} waveforms generated using a *33500B waveform generator* by *Keysight Electronics*. Waveform detection of $\hat{V}_{out}$ was done using a *Wavesurfer 64Xs* oscilloscope by *Teledyne LeCroy*.

The *LXRW19V201-058* varactor is controlled with voltages from 0 V to 5 V and has a capacitance of 200 pF $|_{V_{con}=0\,V}$ and 100 pF $|_{V_{con}=5\,V}$. The parameters selected for the first measurements are as shown in Table 1. At first target values for $\hat{V}_{in}$ and f_{in} are set. These values are then verified via measurement by setting the control voltage to its initial value of $V_{con} = 2.5$ V. The discrepancy in targeted and actual parameters is due to ohmic losses between output of the waveform generator and system input.

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Tab. 1: Measurement parameters for reference measurements

Parameter	set Value	verified Value
$\hat{V}_{in}$	500 mV	489 mV
f_{in}	1000 kHz	900 kHz
C_s	150 pF	

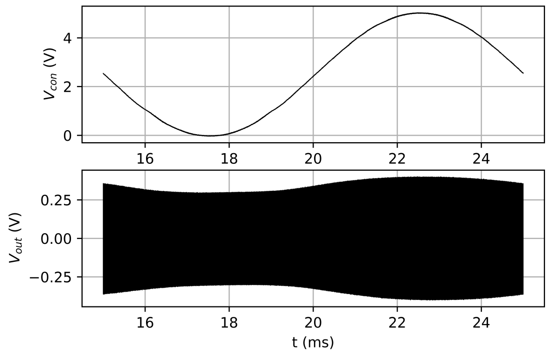

Fig. 4: Measured control voltage V_{con} and RC output voltage V_{out} showing the correlation between control voltage level and output amplitude for sinusoidal input.

Capacitance

At first, a sinusoidal control voltage between 0 V and 5 V at $f_{con} = 100$ Hz is applied as control voltage to verify the capacitance measurement. Figure 4 shows the measured control and output voltage over time. The envelope of the measured voltage $\hat{V}_{out}$ follows the control voltage.

To verify the capacitance measurement, the capacitances at maximum and minimum value of the control voltage are determined and listed in Table 2.

These measurements match the typical characteristic provided by the manufacturer with an error of $2.69\,\%$ and $5.73\,\%$, respectively. This difference can be caused either by inaccuracies in the measurement setup, as for example a slight mismatch in setting the corner frequency, or in component tolerances, as no information about tolerances are provided by the manufacturer. This shows that the measurement setup provides an easy to implement and quick way of measuring capacitance ranges.

Frequency Response

The second measurement is aiming at measuring the response time of the variable capacitance to changes in the control voltage. During this measurement, the same system parameters as given in Table 1 were used. However, instead of a sinusoidal wave a rectangular waveform is applied.

Tab. 2: Results of reference measurements for maximum and minimum capacitance

t	$\hat{V}_{con}$	$\hat{V}_{out}$	a	C
17.54 ms	0 V	299 mV	1.298	194.63 pF
22.51 ms	5 V	401 mV	0.698	104.68 pF

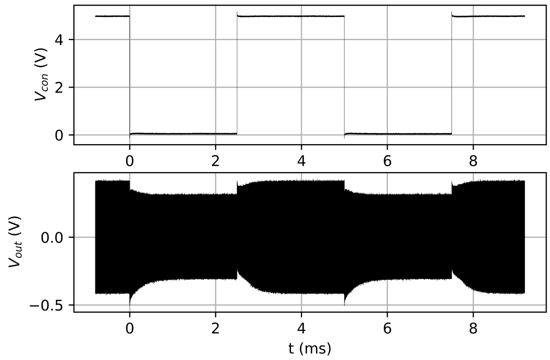

Fig. 5: Measured control voltage V_{con} and RC output voltage V_{out} showing the correlation between control voltage level and output amplitude for rectangular input.

Tab. 3: Expected and measured output voltage amplitude and capacitance at $f_{in} = 757.88$Hz

case	C_{exp}	$\hat{V}_{out,exp}$	$\hat{V}_{out,meas}$	C_{meas}
(16)	181.56 pF	311 mV	326 mV	167.7 pF
(17)	117.95 pF	384 mV	369 mV	130.4 pF

Figure 5 shows the measured control and output voltage over time. The measurement data shows that the capacitance of the varactor does not follow the control voltage instantaneously. The amplitude of the output voltage changes with a time constant of approximately τ =210 µs when modelled by a first-order low pass filter. Using this, the cutoff frequency can be determined to be f_m=757.88 Hz using (18).

This result is verified using a sinusoidal input voltage and the conditions for C specified in (16) and (17). From the conditions and using the values from Table 2 the capacitance values and voltage amplitudes at corner frequency can be calculated and then verified via measurement. The results are shown in Table 3.

The measured capacitance at f_{in} =757.88 Hz is smaller than the expected capacitance, indicating that the DUT's corner frequency f_m must be smaller than 757.88 Hz. Using the direct measurement of the signal amplitude the corner frequency was determined to be approximately 435 Hz.

Conclusion

In this paper an easy to implement measurement methodology was proposed that allows measuring the frequency response of variable capacitances to changes in the control voltage. The advantages of the RC element based measurement method is the comparatively easy implementation using a single resistor, a continouos voltage supply, an oscilloscope, and an arbitrary

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control voltage generator. For adaptation of the DUTs tuning ratio an additional known capacitance can be added. The measurement methodology was verified by measurement of a commercially available ceramic variable capacitance.

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