2D-Photothermal super resolution with sparse matrix stacking

J. Lecompagnon¹, S. Ahmadi², P. D. Hirsch¹, M. Ziegler¹
¹Bundesanstalt für Materialforschung und -prüfung, 12200 Berlin, Germany
Julien.Lecompagnon@bam.de

Summary:
Thermographic super resolution techniques allow the spatial resolution of defects/inhomogeneities below the classical limit, which is governed by the diffusion properties of thermal wave propagation. In this work, we report on the extension of this approach towards a full frame 2D super resolution technique. The approach is based on a repeated spatially structured heating using high power lasers. In a second post-processing step, several measurements are coherently combined using mathematical optimization and taking advantage of the (joint) sparsity of the defects in the sample.

Keywords: super resolution, laser thermography, nondestructive testing, laser scanning, photothermal imaging

Introduction
Photothermal super resolution (SR) is based on a combination of an experimental scanning strategy and a numerical optimization, which has been proven to be superior to standard thermographic methods in the case of one-dimensional linear defects. Due to complexity constraints, laser scanning SR techniques have been mostly limited to evaluation of one-dimensional defect patterns and/or small Regions of Interest (ROI) [1, 2, 3]. Extending the SR problem to more dimensions significantly increases the amount of measurement data and the number of measurements required to achieve sufficient defect resolution to cover large areas. With the incorporation of a limited number of priors, such as a sparse representation of the defect density, and with a purposeful exploitation of the sparse nature of the underlying physical models, the increased complexity can be made manageable.

Methods
The surface temperature of a thin plate exposed to a heating $Q$ with spatial structure $I_{x,y}$ and temporal structure $I_t$ can be described by:

$$T_{\text{meas}}(x,y,z = 0,t) = T_0 + \Phi_{\text{PSF}}(x,y,t) \ast_{xy} a(x,y)$$

$$\Phi_{\text{PSF}}(x,y,t) = \frac{2 \cdot Q}{c_p \rho (4\pi at)^{3/2}} \cdot e^{-\frac{(x-x_0)^2 + (y-y_0)^2}{4at}}$$

$$a(x,y) = a_0(x,y) \ast_{xy} I_{x,y}(x,y)$$

where $T_0$ denotes the initial system temperature, $\rho$ the mass density, $c_p$ the specific heat, $\alpha$ the thermal diffusivity, $(x_0, y_0)$ the coordinates of the centroid of the excitation, $R$ the thermal wave reflexion coefficient ($R \approx 1$), $d$ the plate thickness and $a_0$ the heat source distribution. The operators $\ast_{xy}$, $\ast_t$ represent the convolution operator in the indicated dimensions [2].

The spatial and temporal dimensions can be discretized as follows:

$$x_i = i \cdot \Delta x, \quad y_j = j \cdot \Delta y, \quad t_k = k \cdot \tau_{\text{cam}}$$

$$i \in \{1, \ldots, n_x\}, \quad j \in \{1, \ldots, n_y\}, \quad k \in \{1, \ldots, n_t\}$$

A series of $m \in \{1, \ldots, n_m\}$ independent measurements can be described by:

$$T_{\text{meas}}[x_i,y_j,t_k,m] = T_0 + T_{\text{diff}}[x_i,y_j,t_k]$$

with $T_0$ representing the initial temperature of the sample at $t = 0$ s and $T_{\text{diff}}$ being the differential temperature caused by thermal excitation. In order to reduce the problem complexity, the time dimension can be eliminated by choosing a timestep $t = t_{\text{eval}}$ and only take the temperature change with respect to $T_0$ further into account:

$$T_{\text{diff}}[x_i,y_j,m] = T_{\text{meas}}[x_i,y_j,t = t_{\text{eval}},m] - T_0$$

The spatial dimensions can then be merged by flattening to a single dimension $r$, applying a bijective transform assigning an index $r \in \{1, \ldots, n_x \cdot n_y\}$ to every pixel coordinate $[x_i, y_j]$: $T_r[n,m] = T_{\text{diff}}[x_i,y_j,m]$.

The inverse transform of Eq. (10) can be applied to reshape the data back to a two-dimensional image. As an approximative model, the defect response can be defined as the convolution of the thermal PSF as a Green’s function kernel and the heat source distribution $a_r[n,m]$ [4]:

$$\Phi_{\text{PSF},r}[n] \ast_r a_r[n,m] = T_r[n,m]$$

The single measurement solutions can then be merged by summation.

$$a_{\text{rec}}[n] = \sum_m a_r[n,m]$$
To efficiently solve Eq. (11), it can be transformed to a multiplicative problem by introducing the discrete convolution matrix $h(PSF)$, such that:

$$h(\Phi_{PSF,F}^r) \cdot a_p^m = \begin{bmatrix} 0 \\ T_{m_0}^r \\ 0 \\ 0 \end{bmatrix} = T_{r_0}^m$$  \hspace{1cm} (13)

with dimensions $\Phi_{PSF,F}^r \in \mathbb{R}^{n \times n_r}$, $h(\Phi_{PSF,F}) \in \mathbb{R}^{[2(n_y-1) \times n_x \times n_y - 1]}$, and $T_{r_0}^m \in \mathbb{R}^{2(n_y-1) \times n_{m}}$. The convolution matrix $h$ is a sparse lower triangular matrix with Toeplitz-structure, allowing it to be stored memory-efficient despite its large dimensions.

This leads to solving $n_m$ multiplicative inversion problems. In order to be able to exploit the joint sparsity between measurements in all $n_m$ equations, they need to be solved simultaneously. This can be achieved by stacking:

$$H \cdot A = \begin{bmatrix} h & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & h \end{bmatrix} \begin{bmatrix} a_1^m \\ \vdots \\ a_m^m \end{bmatrix} = \begin{bmatrix} T_{r_0}^1 \\ \vdots \\ T_{r_0}^m \end{bmatrix} = T_{r_0}$$  \hspace{1cm} (14)

with dimensions $H \in \mathbb{R}^{(2n_y-1) \times n_{n_m} \times n_{m}}$ and $T_{r_0}^m \in \mathbb{R}^{(2(n_y-1) \times n_{m}}$. $H$ is a sparse diagonal block matrix, which makes it efficient to store.

To enhance sparsity even further, a threshold is applied where

$$\lambda_2 \approx \phi_{PSF,F}$$

with $\phi_{PSF,F}$ the front surface. Each defect has an edge length of 2 mm with decreasing distances within the pairs.

220 blind measurements with randomly sampled excitation positions across the ROI with a laser power of $P = 15$ W, a pulse duration of $\tau_{on} = 0.2$ s and a laser spot size of $d_{spot} = 1.5$ mm have been conducted. Applying our previously described 2D-SR evaluation technique, the resulting defect reconstruction $a_{rec}$ is displayed in Fig. 2.

For reference, a single measurement with homogeneous illumination across the sample surface has been performed. The measured data is shown in Fig. 3.

Fig. 2 Resulting defect density $a_{rec}(x,y)$ from solving Eq. (15) with an ADMM penalty $\rho_{ADMM} = 16$, $\lambda_2 \approx 375$, $\lambda_2 = 20$ for $\tau_{on} = 0.3$ s. The green boxes indicate the defect position and sizes. All defects and even close defect spacings up to 0.5 mm are resolved.

Fig. 3. Thermogram for full area homogeneous illumination for reference. $P = 450$ W, $\tau_{on} = 0.5$ s sampled at $\tau_{eval} = 0.5$ s. All defects are clearly visible but closer defects cannot be resolved independently.

References


