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What did the current SI do to the uncertainties in measurement of defining constants?

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Summary:

We present a brief axiomatic description of the current SI. A set of seven defining fundamental constants were assigned an exact value in the SI based on their best-known quantity values in terms of the previous SI units. The defining constants so established redefine the SI base and other units. The previous uncertainties in measurement of the defining constants did not evaporate. They were shifted to the physical realizations of the SI units of related quantities.

Keywords: Avogadro number, Kilogram, Planck constant, Uncertainty, Unit of measurement

Introduction

The current SI (ninth edition, dated 2019) has established new definitions for the SI base units by fixing the numerical values for a chosen set of seven fundamental or technical constants of nature referred to as the defining constants. The base units of the SI are still second (s), meter (m), kilogram (kg), ampere (A), kelvin (K), mole (mol), and candela (cd). The seven defining constants chosen to redefine the base units are the hyperfine transition frequency of the 133-cesium atom ($\Delta \nu_{\mathrm{Cs}}$), the speed of light in vacuum (c), the Planck constant (h), the elementary charge (e), the Boltzmann constant (k), the Avogadro constant (N_{A}), and the luminous efficacy of monochromatic radiation of frequency 540 × 10^{12} Hz (K_{cd}). The best-known quantity values in terms of the previous SI units were assigned to the defining constants. Informally, the measurement uncertainties associated with the assigned quantity values were zeroed. The quantity values of the defining constants so established redefine the SI units [1].

In the SI, a metrological expression for a quantity value Q is a product of a number {Q} and a unit of measurement [Q]. That is, Q = {Q} [Q]. The quantity is compared with a physical realization of the unit [Q] to determine the number {Q}. The number {Q} is uncertain due to imperfections of measurement (including incomplete description of the quantity measured). For most applications, this uncertainty is much larger than the uncertainty in the physical realization of the SI units.

In the current SI, seven defining constants were chosen as the quantity Q*. A suitable number {Q} was determined for each defining constant. Then the ratio Q*/{Q} was set as the revised unit [Q] for that defining constant [1]. The suitable number for each defining constant was determined such that the current SI units are backward compatible with the previous SI units.

An SI unit of measurement must be (1) a constant, (2) backward compatible with the previous SI units to maintain continuity, (3) and convenient for physical realization to develop measurement standards. The backward compatibility to previous units subsumes (i) that the units are of practical size, and (ii) that the SI is an interconnected system of coherent units. Physical realizability was an important consideration in the choice of defining constants. The chosen defining constants allow for practical realizations with smallest uncertainties. In principle, defining constants are available to everyone and at all times. A link to special artifacts is not needed [1].

The defining constants are unique invariant quantities. So, a defining constant can, in principle, be used as a unit of measurement for quantities of the same kind. As potential units, the defining constants are either too small or too large for practical use. So, a practical unit of measurement [Q] would be proportional to the defining constant Q*; that is [Q] = κ Q* for some constant of proportionality κ. A fundamental requirement is that a revised definition must be backward compatible with the previous SI units. Suppose {Q} is the numerical part of the established value of a defining constant Q* in terms of the previous SI unit. Consider the following definition of a unit of measurement

$$[Q] = \{Q\}^{-1}\, Q^* \qquad (1)$$

where $\{Q\}^{-1}$ is the constant of proportionality. The equation (1) can be written as

$$Q^* = \{Q\}\, [Q] \qquad (2)$$

The equation (2) is profound for it states that the magnitude of the SI unit [Q] is such that the established value {Q} [Q] is exactly equal to the defining constant Q* (quantity). This is the foundation of the current SI based on the established values of seven defining constants. Per equation (1), the SI units based on defining constants are unique invariant quantities (constants). Per equation (2), the SI units based on the defining constants are backward compatible with the previous SI units. Table 1 is reproduced from the current (ninth) edition of the SI Brochure [1].

Table 1. The seven defining constants of the SI and the seven corresponding units they define.

Defining constant	Symbol	Numerical value	Unit
hyperfine transition frequency of Cs	$\Delta\nu_{\mathrm{Cs}}$	9 192 631 770	Hz
speed of light in vacuum	c	299 792 458	$\mathrm{m\ s^{-1}}$
Planck constant	h	$6.626\ 070\ 15 \times 10^{-34}$	J s
elementary charge	e	$1.602\ 176\ 634 \times 10^{-19}$	C
Boltzmann constant	k	$1.380\ 649 \times 10^{-23}$	$\mathrm{J\ K^{-1}}$
Avogadro constant	N_{A}	$6.022\ 140\ 76 \times 10^{23}$	$\mathrm{mol^{-1}}$
luminous efficacy	K_{cd}	683	$\mathrm{lm\ W^{-1}}$

The symbols Hz, J, C, lm, and W represent the units hertz, joule, coulomb, lumen, and watt, which are defined as $1\ \mathrm{Hz} = 1\ \mathrm{s^{-1}}$, $1\ \mathrm{J} = 1\ \mathrm{kg\ m^2\ s^{-2}}$, $1\ \mathrm{C} = 1\ \mathrm{A\ s}$, $1\ \mathrm{lm} = 1\ \mathrm{cd\ m^2\ m^{-2}}$, and $1\ \mathrm{W} = 1\ \mathrm{kg\ m^2\ s^{-3}}$, respectively. The column 1 lists the names of the defining constants (quantities). The symbols in column 2 are for both the defining constants and their established values. The products of the numerical values in column 3 and the corresponding units in column 4 are the established values of the defining constants. The revised SI units are of such magnitude that the defining constants are exactly equal to their established values. The defining constants divided by their numerical values are the current definitions of their SI units. These SI units of defining constants are algebraically solved to obtain the current definitions of the SI base units (s, m, kg, A, K, and cd), and the other SI units given in the current SI Brochure [1]. The hyperfine transition frequency of the 133-cesium atom, the speed of light in vacuum, and the luminous efficacy were established in 1967, 1975, and 1979, respectively [1]. Thus, the world was already using the definitions of the second, the meter, and the candela that were based on establishing the values of defining constants. The current SI updated earlier definitions.

Previous uncertainties in measurement of defining constants were shifted to physical realizations of the units of related quantities

The SI is an inter-connected system of units. When the uncertainty associated with one quantity value changes, the uncertainties in the related quantity values also change. The zeroing of uncertainties in measurement of defining constants to establish their values had the consequence of shifting those uncertainties to physical realizations of the units of related quantities [2]. The 2017 recommended value of the Planck constant h, for example, had a relative standard uncertainty of 1×10^{-8}. In the revised SI, this uncertainty was shifted to the mass of the international prototype of kilogram (IPK). As of 2019, the mass of the IPK has quantity value 1 kg with a relative standard uncertainty of 1×10^{-8} [1]. The 2017 recommended value of the molar Planck constant $N_{\mathrm{A}}h$ had a relative standard uncertainty of 4.5×10^{-10}. In the current SI, this uncertainty was shifted to the molar mass of carbon 12, $M(^{12}\mathrm{C})$. As of 2019, $M(^{12}\mathrm{C})$ has quantity value 0.012 kg/mol with a relative standard uncertainty of 4.5×10^{-10} [1]. The relative standard uncertainty associated with the 2017 recommended value of the Boltzmann constant k was close to 3.7×10^{-7}. This uncertainty was shifted to the Triple point of water, T_{TPW}. As of 2019, T_{TPW} has quantity value 273.16 K with a relative standard uncertainty of 3.7×10^{-7} [1]. In the future these uncertainties will be determined experimentally.

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References

[1] The International System of Units (SI), 9th edition (2019); https://www.bipm.org/utils/common/pdf/si-brochure/SI-Brochure-9-EN.pdf

[2] CODATA recommended values of the fundamental physical constants: 2018; https://doi.org/10.1103/RevModPhys.93.025010