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SI Traceable Small Force Generation and Measurements via Photon Momentum

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Summary:

In this contribution we present the concept of photon momentum enabled SI-traceably made small force generation and measurements below the conventionally accepted limits. The developed instrumentations, the measurement infrastructure and the obtained results demonstrate the advantages of this concept and further are extended to present the means of systematization of the force measurements covering the range below 10 µN to several tens of nN. The prospects to reduce the relative measurement uncertainties of small force and small weight measurements are discussed.

Keywords: Photon momentum, small force, small weight, laser power, Planck-balance.

Background

Since the year 2019 the unit of the mass, the kilogram, in the International System of Units (SI) [1] is defined by a natural constant, namely through the fixed numerical value of the Planck constant. According to the present definition, the value of the Planck constant is h= 6.62607015×10^{-34} Js, expressed in base units kg m^2s^{-1} [1]. There exist two well established methods for practical realization of the kg. One is based on counting the atoms of a silicon sphere by X-ray crystal density method (based on concept of the inertial mass) and later disseminating the value using gravitational mass measurements. The other is the Kibble balance (KB) method [2] that exploits the gravitational force (gravitational mass) and compares the effective compensation force measured as a mechanical power with the electrical powers of the sensor obtained from a two-step experiment both steps based on electromagnetic interaction and with direct traceability to macroscopic quantum effects: the Josephson effect and the quantum Hall effect. Until now the primary realization and subsequent key-comparisons are made only for 1 kg. All other smaller values are still obtained using conventional accepted standard methods and instrumentations. Improvements/drawbacks have not been seen while undergoing this big change in mass metrology. Since the SI traceable calibrated force measurements are directly connected and are referenced with the mass values including for very small values, therefore in force metrology no substantial changes are noticeable as well. The KB allows to realize methods and determining a mass of any value in terms of the Planck constant without the use of any other mass standard including for lowest levels (>1 mg) and for any arbitrary value (e. g. 3.247 g) directly without the need of interpolation between standard mass values (e. g. 1 g, 2 g, 5 g, 10 g) and for all other derived units such as force, torque, etc. Therefore, a new class of the specially designed apparatuses would potentially simplify the calibration procedures and minimize the necessary time and, as a consequence, the respective economic burden. There exists already a table-top version of KB, e.g. Planck-Balance 2 (PB2) [3], an apparatus that allows SI traceable instrumentation based standard mass calibrations from 100 g down to 1 mg with measurement uncertainties corresponding to the weights of E2 class in air following OIML R 111-1 [4]. At 1 mg the typical uncertainties are about 0.3% and it grows to high % as the scale reaches to µg level due to very well described material and instrumentation limitations.

Photon momentum method

A complementary method using the photon momentum generated small forces offers powerful means to test and to reduce the uncertainty of the measurements and characterize instrumentations in a SI-traceable manner. This method relies on the option to reference the measured small forces in relation with the magnitude of the measured optical power of lasers in accordance to the

$$F = \frac{Power}{c}(1 + R_L)\cos\theta \qquad (1)$$

where Power is given by calibrated optical detector, c is the speed of the light, R_L is the reflectivity coefficient of the mirror on which the force is generated while laser is impinging and reflecting from it. The force exerted by a CW laser source with 1.5 W average optical power is equal to 10 nN, which is equivalent to the gravitational (*g*) force acting on the approx. 1-µg-mass piece, to be determined as

$$F = mg \tag{2}$$

If highly reflective and well-characterized mirror is used, a multiple reflection can be created with negligible optical power loses by which amplification of forces can be achieved as

$$\begin{gathered}\sum_{i=1}^{N} F_i = \frac{(1+R_L)}{c}\sum_{i=1}^{N} Power_i,\\ \sum_{i=1}^{N} \mathrm{Power}_i = \mathrm{Power}_1 \sum_{i=1}^{N} {R_L}^{i-1}\end{gathered} \tag{3}$$

For example, a reference force of appox. 10 µN (1 mg) can be generated with 100 W power and 15-reflections. As a result, a short SI traceability chain can be constructed with minimal uncertainty contributing parameters, i.e. combined uncertainty of the optical power detector and the reflectivity value of the mirror [5]. Combining eqs. 1 and 2 yields

$$m = Power\,\frac{1+R_L}{c\cdot g} \tag{4}$$

$$\frac{u(m)}{m} = \sqrt{\left(\frac{u(\mathrm{Power})}{\mathrm{Power}}\right)^2 + \left(\frac{u(R_L)}{R_L}\right)^2 + \left(\frac{u(g)}{g}\right)^2} \tag{5}$$

The value of *u(g)/g* can be determined by means of a (free-fall) absolute gravimeter to approximately 0.2 ppm and below better than 0.01 ppm. The values of $u(R_L)/R_L$ for the ultra-high reflective mirrors in accordance with most datasheets provided by different manufacturers varies in the range of 10 ppm to 70 ppm. The *u(*Power*)/*Power typically varies dependent from absolute magnitude of the applied laser. For example, in accordance with PTB provided calibration services for the detector calibration in reference with primary standard it is approximately 0.1%-0.5% for 100 W and by the use of state-of-the-art cryogenic primary standards as low as nW orders of powers can be detected, with the upper limit typically given for below 1 mW power level with an expanded measurement uncertainty of about 0.002 %.

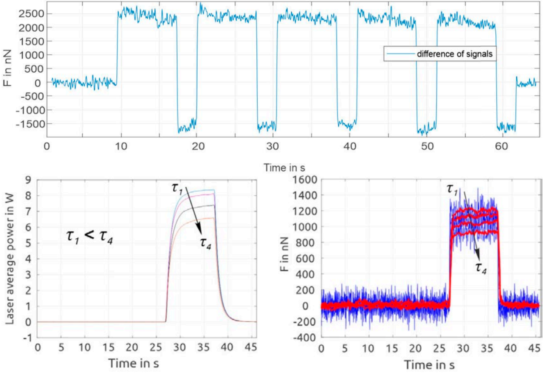

Figure 1. Measured forces via photon momentum and referenced by input laser power. [6]

The *u(m)/m* known from conventional mass metrology, from the ‘uncertainties of the weights of the classes E1, E2, F1, and F2 according to OIML R111’ is not specified below 1 mg that has already 0.3 % error limits (permissible tolerances). Thus, if the photon momentum is used for generation of forces referenced by high-precision SI-traceable conventional measurement methods and converted to mass values, then better uncertainty can be obtained both practically and by computations (eq. 5).

In practice, at Institute of Process Measurement and Sensor Technology in TU Ilmenau, such measurements are already realized for the force generation below this 1 mg (10µN) limits (Fig.1). In upper panel up-to-now measured maximum forces generated by the input high-power pulsed lasers of 17 W level at 33-reflection configuration with 75% duty cycle (10s) operation of periodic on-and-off signal is presented. In lower panel, the input laser power during each 10s is modulated by tuning the pulse width at 250ns, 1.5µs, 5µs, 10µs, and the corresponding force measurements in case of 21-reflections are shown. The current progress in referencing the photon momentum generated forces via optical power measurements of the input laser is limited to below 0.5% due to yet existing minor technical implementation problematics, e.g. proper choice of the laser and high-reflectivity mirror that should be optimized for the lasers’ wavelength.

Outlook

Future steps in these developments are directed to implement a comprehensive uncertainty analyzes of the force measurements and scale calibration using a special apparatus known as Photon momentum setup that utilizes at the same time the principle of the KB similar to PB2 setup [6]. With the setup, preliminary, an uncertainty of approximately 0.1% for 10 µN, 1 µN, 100 nN force measurements referenced by photon momentum generated forces are expected.

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