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Uncertainty in data fusion of coordinate measurements (Unsicherheit bei der Datenfusion von Koordinatenmessungen)

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1 Data fusion tasks and realizations in coordinate metrology

The complexity of the tasks in coordinate metrology is increasing and the requirements concerning the measurement time and uncertainty are becoming more stringent. This makes a measurement strategy with a single setup and a single measurement not always possible. Measurement object characterization often requires a multi-sensor and a multi-orientation measurement strategy that involves multiple view measurements as well as multiple probing. The process required to combine data from different sources defined as data fusion [1].

The goal of data fusion is data collecting or data improving (Fig. 1) to obtain *new* or *more precise* knowledge about physical quantities [2]. Being used in a wide range of measurement processes, data fusion allows increasing effectiveness, enhancing scope and applicability, increasing accuracy and precision of the measurement. The task of data fusion also defines the configuration of the sources: competitive, complementary or cooperative [3].

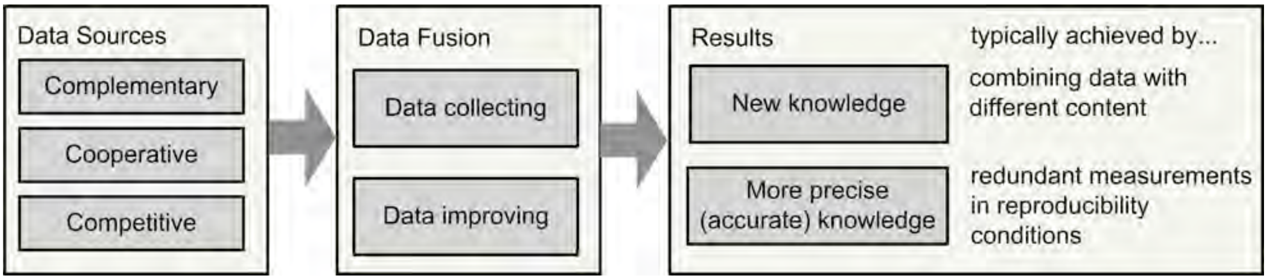

Fig.1. Data fusion in metrology.

There are three levels of data fusion (classification by output): raw-data level, feature level and decision level [1, 2]. Typical for coordinate metrology is the fusion of point clouds implemented on the low level (raw-data level) of data fusion. In fusion of coordinate measurements data the *registration* process is crucial. It is needed to transfer all measured points to a common coordinate system. The registration is realized either by the capabilities of the precision positioning or by matching of overlapping areas of the point clouds. The registration process is studied in this paper.

2 Measurement uncertainty evaluation

The two main approaches to the uncertainty evaluation in coordinate metrology are: the experimental and the model-based approaches. The first one is based on the use of calibrated workpieces or measurement standards (ISO 15530-3:2011). *Experimental* approach (substitution method) is only valid for measuring workpieces that are nominally identical to the reference artifact used, measured in the same location and using the same measurement strategy [4].

The *model-based* approach uses:

- the GUM uncertainty framework (GUF), which provides an implementation of uncertainty propagation in linear approximation;
- Monte-Carlo method (MCM) providing an implementation of propagation of distributions (ISO/TS 15530-4:2008).

A measurement model in the basic document about measurement uncertainty GUM [5] is suitable for the one-dimensional output and implicit model function. In coordinate metrology the output is usually multi-dimensional and the model is not given by implicit function. GUM Supplement 2 [6] treats multivariate measurement models. They are generally characterized by mutually correlated outputs because they depend on common input quantities.

A multivariate measurement model $\boldsymbol{h} = (h_1 \ldots h_m)^T$ in vector representation of the general form is:

$$\boldsymbol{h}(\boldsymbol{Y}, \boldsymbol{X}) = 0 \,, \quad (1)$$

where $\boldsymbol{X} = (X_1 \ldots X_n)^T$ - input vector quantity; $\boldsymbol{Y} = (Y_1 \ldots Y_m)^T$ - output vector quantity.

Uncertainty in multidimensional case is described by: joint distribution, covariance matrix or coverage region. The examples of simulated joint distributions are given in Fig. 2. In coordinate metrology one often deals with non-Gaussian distributions. These distributions can be simulated by a copula, which is the function that describes the dependence between random variables. It can be used to produce the *joint distribution* for the different marginal (one-dimensional) distributions [7]. *Coverage region* can be presented in two forms: either as a hyper-ellipsoid or a hyper-rectangle (dimension corresponds to the number of output quantities).

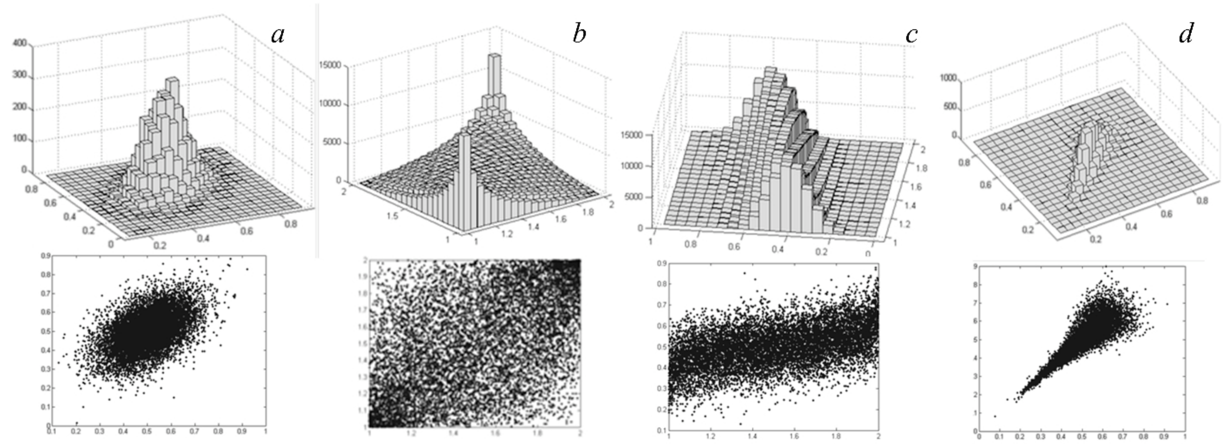

Fig. 2. Examples of joint distributions (two-dimensional) simulated by Monte-Carlo method: (a) Gaussian bivariate distributions; (b) Gaussian copula, marginal distributions are uniform; (c) Gaussian copula, marginal distributions are uniform and Gaussian; (d) Clayton copula, marginal distributions are Gaussian.

GUF-based approach operates with *covariance matrices* $U_{\boldsymbol{Y}}, U_{\boldsymbol{X}}$:

$$U_{\boldsymbol{Y}} = C U_{\boldsymbol{X}} C^T, \, C = C_{\boldsymbol{Y}}^{-1} C_{\boldsymbol{X}} \,,$$

$$U_{\boldsymbol{X}} = \begin{bmatrix} u^2(x_1) \ldots u(x_1, x_N) \\ u(x_N, x_1) \ldots u^2(x_N) \end{bmatrix}$$

where $C_{\boldsymbol{Y}}, C_{\boldsymbol{X}}$ are the sensitivity matrices (partial derivatives).

The Monte-Carlo method (GUM Supplement 1) allows the measurement uncertainty estimation for non-Gaussian random variables and for significantly non-linear models. MCM is also simple to implement. To compare both approaches, the following example of trilateration method is considered (Fig. 3). The coordinates in a Cartesian coordinate system are evaluated from the three distance measurements d_1, d_2, d_3. This is a case of non-redundant indirect measurements. For redundant measurements with more than three distances (multilateration), the over-determined system of equations has to be solved by the least squares method (LSM). In practice, usually four sources (for example, lasertracers) are applied to provide autocalibration [8].

Measurement model according (1):

$$\boldsymbol{h}_i \, (\boldsymbol{Y}, \boldsymbol{X}) = (\boldsymbol{x} - \boldsymbol{x}_{0i})^2 + (\boldsymbol{y} - \boldsymbol{y}_{0i})^2 + (\boldsymbol{z} - \boldsymbol{z}_{0i})^2 - \boldsymbol{d}_i^2 = 0, \;\; \boldsymbol{i} = 1, 2, 3.$$

where: $\boldsymbol{X} = (x_{0i}, y_{0i}, z_{0i}, d_i)$ - vector of input quantities; $\boldsymbol{Y} = (x, y, z)$ - vector of output quantities. Since analytical derivations for the GUF application are quite cumbersome, the Matlab Symbolic Toolbox was applied.

In a case of large relative uncertainties of the distance measurement, the non-linearity of the measurement model becomes significant and adds some asymmetry to the distribution of the measurement uncertainty of evaluated coordinates (see Fig. 3c).

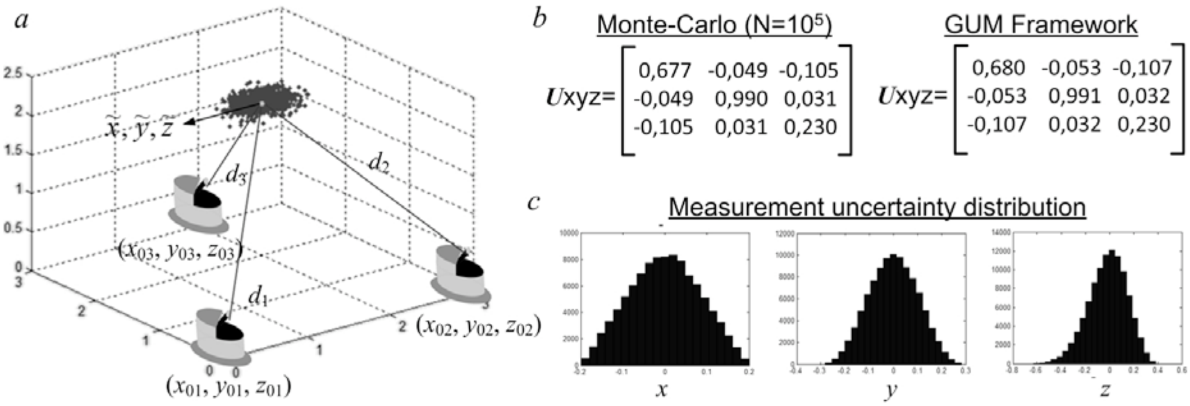

Fig. 3. Examples of measurement uncertainty evaluation in a case of trilateration: (a) scheme and simulation by using Monte-Carlo method; (b) covariance matrices evaluated by MCM and GUF for an example with measurement uncertainties of distances $u(d_i)$ = 0,01; (c) histograms for the distributions of measurement uncertainties of coordinates in the case of significant nonlinearity, input distributions are simulated as uniform.

Measurement models in coordinate metrology are usually *multi-stage*, where the output from one stage becomes the input to the next. We deal with large-volume data and optimization algorithms, which can result in a longer calculation time. Nevertheless Monte-Carlo simulations are preferable in such cases due to the high complexity of analytical modeling for the GUF implementation.

3 Registration

After single measurements are carried out, the results are to be fused. Each point cloud is presented in its own coordinate system. Coordinate transformations are required to obtain all measurement points in a common coordinate system, e.g.: point clouds in the coordinate system of the instrument have to be transformed to the workpiece coordinate system. The transformation of coordinates causes a transformation of the measurement uncertainty. For the following analysis we differentiate a transformation between different orthogonal coordinate systems and a transformation within one coordinate system.

3.1 Transformation to another coordinate system

Although a commonly used coordinate system is the Cartesian or rectangular coordinate system (x, y, z), other systems have an application in measurement practice: spherical or polar coordinates (ρ, θ, φ) are well adapted to the symmetry of laser trackers; while cylindrical coordinates (ρ, θ, z) are recommended to be used for data processing of form measurements of rotationally symmetric objects (Fig. 11, [9]). A transformation from one orthogonal coordinate system to another is usually non-linear (Fig. 4). This has to be taken into account when GUF is used.

3.2 Transformation in Cartesian coordinate system

Data processing of point clouds involves coordinate transformations: for alignment and for transformation of the point clouds to the same coordinate system. For registration the following should be assessed: (I) choice of the matching method and the corresponding optimization criterion; (II) the transformation matrix to perform an appropriate transformation (similarity, affine, polynomial, bilinear, projective etc.), a number of transformation parameters; (III) a reference coordinate system to allow an easy alignment and to reduce the uncertainty caused by the transformation of the coordinates. Registration can be carried out by: data of system position (for example in CMM); matching of overlapping areas (point-to-point or feature-based algorithms).

Features used for registration are classified as natural and artificial. Examples of natural features are the corners or the edges of the object and the non-idealities of the surface. Typical artificially created features for registration are: a stochastic pattern (Fig. 9b), widely-used circular markers (Fig. 9c) and spherical (3D) fiducial markers.

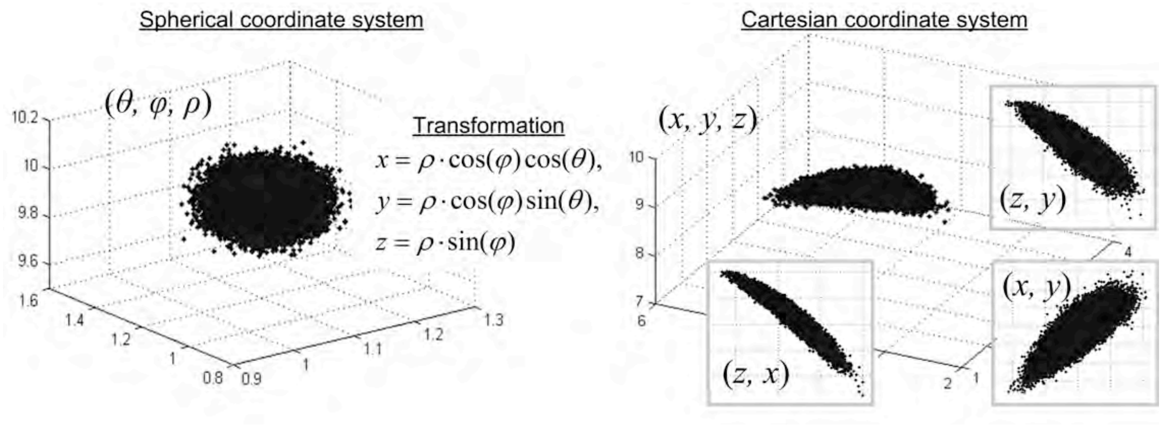

Fig. 4. Uncertainty propagation in transformation from spherical to Cartesian coordinates. Initial uncertainty simulated by Gaussian joint distribution. Due to the nonlinear transformation function and large relative uncertainty, the transformed uncertainties of coordinates have asymmetrical distribution.

The linear transformation in Cartesian coordinate system can be defined by the transformation matrix R (3x3) and the translation vector of the origin $\vec{x}_0$ – similarity transformation (Fig. 5). Different criteria can be used to evaluate R, based on the minimization of the function of the distances (LSM) or the maximization of the cross correlation between elements. However, the transformation matrix is not known exactly and the uncertainty of the transformation parameters has to be taken into account (Fig. 5).

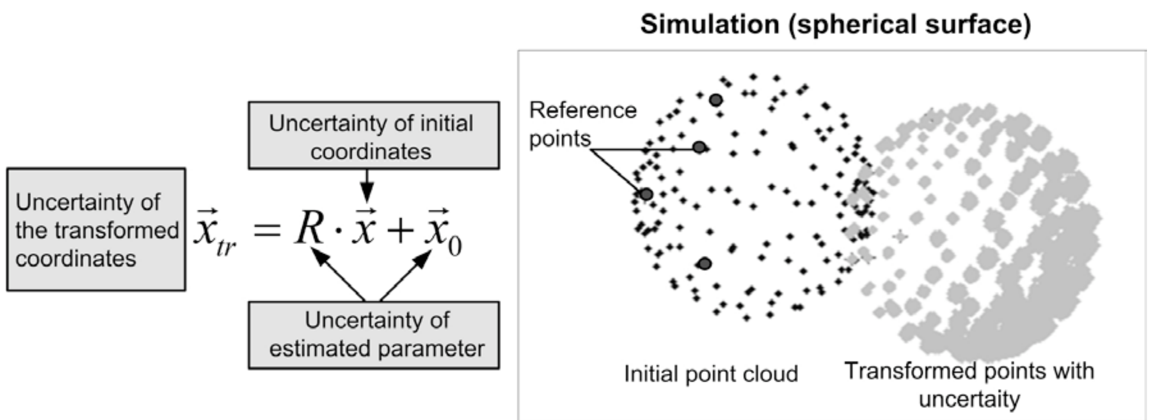

Fig. 5. Uncertainty transformation. Left: linear transformation in Cartesian coordinate system; right: uncertainty of the transformed points on the sphere, the transformation parameters are estimated from the coordinates of the reference points.

The transformation parameters are found (LSM) by the set of reference points in each of the two point clouds. Here we assume that the coordinates of one point cloud are transformed to the coordinate system of the second point cloud. The next sequence of events explains the simulation:

- due to the uncertainty in the estimate of reference points' coordinates, the estimates of the transformation parameters also are evaluated with uncertainty;
- using the evaluated parameters, the coordinates of all points in the first point cloud are transformed;
- uncertainty of the transformation parameters is added to the initial uncertainty of the points from the first point cloud.

Obviously, the uncertainty of the transformation parameters and consequently of the transformed coordinates depends on the number and distribution of the reference points, which are used for transformation parameters calculation. For example, the relative reduction of the measurement uncertainty of the circle radius depending on the number of reference points is studied in [10]. Increasing the number of points from 3 to 6 reduces the uncertainty of the radius estimate by more than 30% (points are homogeneously distributed on the circle).

It is important that even if the uncertainties of the coordinates were independent before the transformation, the transformed coordinates are *correlated*. The reason is the common source of uncertainty. The correlation function is dependent on the uncertainty of the transformation parameters.

4 Reduction of the measurement uncertainty by data fusion

The measurement uncertainty is connected with risks in conformity assessment and if the measurement uncertainty exceeds the target uncertainty (according to the specification, tolerance limits, etc.), it has to be reduced. ISO 14253-2:2011 introduces the Procedure for Uncertainty Management (PUMA) with an iterative method as a tool to maximize profit and minimize cost in metrological activities. According to this procedure, the first step of the uncertainty management is the re-consideration of the uncertainty calculation step (for example of the assumed uncertainty distribution, etc.). Measurement uncertainty modeling is mentioned in Section 2. The next step includes the changes in measurement procedure.

The basic idea of *data improving* by data fusion is the use of *redundant measurements* (instrumental or procedural redundancy) to reduce the measurement uncertainty due to the random and systematic effects. In Fig. 6 multilateration method is analyzed (simulated uncertainty for 3 and 4 distance measurements). Obviously, the increasing number of the measurements decreases the measurement uncertainty. The configuration of the measurement system has significant influence (see Fig. 6 right). Monte-Carlo simulation is used as the basis for the optimization of lasertracers location [8] and can be applied for the markers' location optimization (Fig. 14).

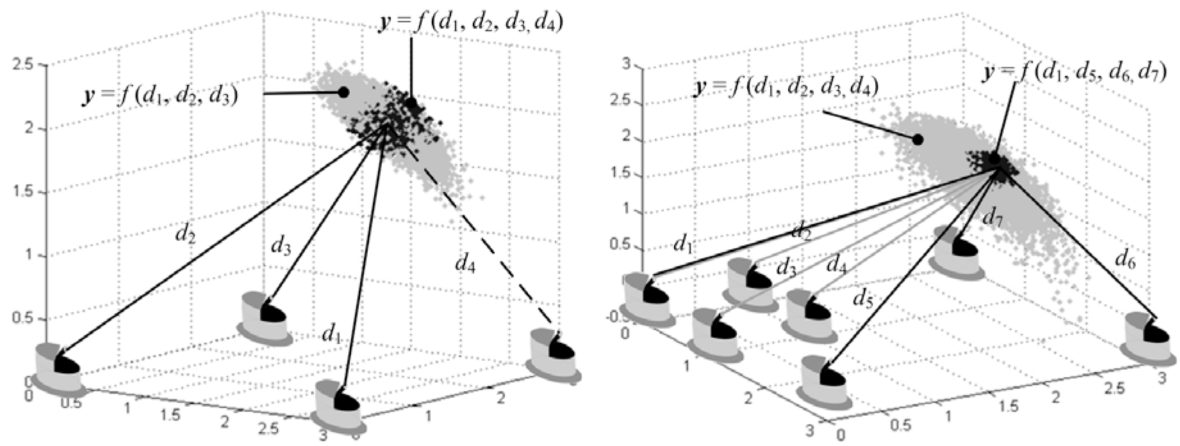

Fig. 6. Measurement uncertainty reduction in a case of multilateration. Left: due to the number of distance measurement, $\boldsymbol{y} = f(d_1, d_2, d_3, \underline{d_4})$ – fourth distance reduces the uncertainty significantly. Right: due to the configuration, reference points (i.e. lasertracers) located homogeneously around measurement point give significantly smaller uncertainty.

Not only instrumental redundancy can be used. In coordinate metrology the different conditions can be used to provide reproducibility conditions: different parameters of sensors, illumination and object location. For example, for image level [11] fusing images of the same scene acquired by different sensors or taken with different illumination or observation parameters: illumination intensity or direction, structured illumination, camera position and orientation, spectral response, focus, etc.

4.1 Mathematical considerations

There are many analytical techniques for data fusion based on theories for representing imperfect data [12]: probabilistic theory (Bayesian approach) (Fig. 7a), possibility theory (fuzzy sets) (Fig. 7b), Dempster-Shafer's theory of evidence, rough sets theory (Pawlak). The commonly used approach based on probabilistic theory in form of weighting procedures. The feature extraction with raw-data level fusion is represented by weighted least-square method, and for the feature-level fusion mostly the weighted mean is used:

$$\widetilde{x}_F = \frac{\sum_{i=1}^{m} \widetilde{x}_i w_i}{\sum_{i=1}^{m} w_i}, \quad u_f = \left(\sum_{i=1}^{m} w_i \right)^{-\frac{1}{2}}, \quad (2)$$

where $\widetilde{x}_i$ – estimate for i^{th} source with standard uncertainty u_i, w_j – weights, u_f – standard uncertainty of the weighted estimate. Properly evaluated weights for the sources define the benefit of the data fusion process. The weighting can be global or local.

As mentioned in Section 2, measurement uncertainty can be represented as a hyper-ellipsoid. Data fusion can be considered as the intersection of ellipsoids representing different data sources (Fig. 7c). As an example, a significant improvement can be achieved in microscopic measurement, where the difference between the lateral and vertical resolution is significant [13]. For the measurement of the object located on the tilting stage, a benefit is achieved by different orientation of ellipsoids, which characterize uncertainty of the coordinates (Fig. 7c). As underlined in Section 3.2, the uncertainty of the registration process has to be taken into account.

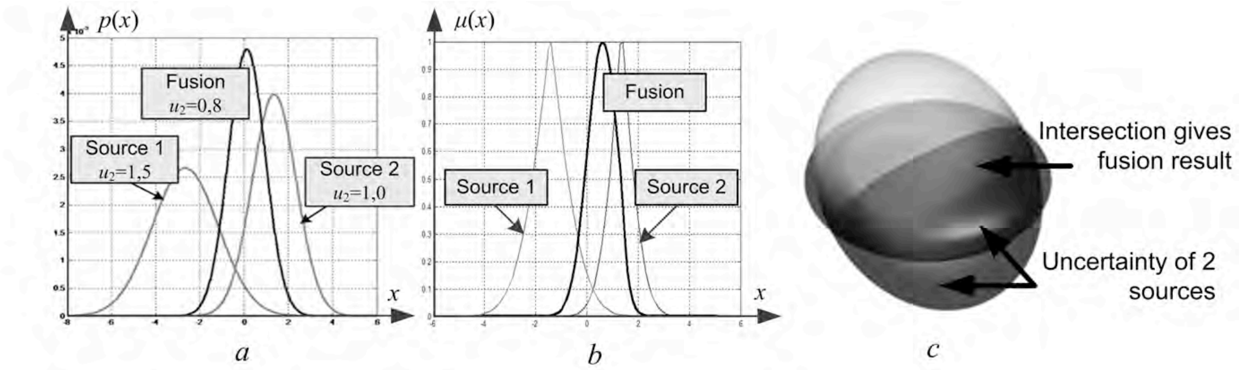

Fig. 7. Representation of data fusion: (a) probability density functions; (b) fuzzy numbers (possibility distribution); (c) 3D fusion as intersection of two ellipsoids, the result can be described by minimum-volume circumscribed ellipsoid.

4.2 Example of multiresolution data fusion

A typical task in coordinate measurements is fusion of data with different point densities and uncertainties. The idea is to obtain high density point cloud and correct the systematic effects by additional high precision measurements with low point density. For example, measurement results from computer tomography are corrected by tactile measurements [14].

In the following simulations, the process of uncertainty simulation for circular models by Monte-Carlo method is presented, where the algorithm of simulation with separation of random and unknown systematic effects is applied [15]. Random effect simulated as a random value for each point. Unknown systematic effects are modeled as the additive error, whose value is the equal for all points and changed randomly for each iteration of Monte-Carlo simulation. This results in a correlation between uncertainties of the points and makes the model more realistic.

The following simulation is carried out. The measurement of the circular features is provided by two sensors with different characteristics. The first sensor provides a larger number of points and the second gives only several points, but with higher accuracy and precision. Data fusion on the feature level is represented by the intersection of the ellipsoids, which characterize the circle parameters (R, x_0, z_0) (Fig. 8).

Another kind of multiresolution fusion for the cylindrical measurements is given in [9]. Helical scan mode measurements allow collecting the data of cylinder surface, which can be corrected by fusion with the high precision diameter measurements on the chosen fifteen Z-levels. In this case, the contributing data sources have significantly different uncertainties.

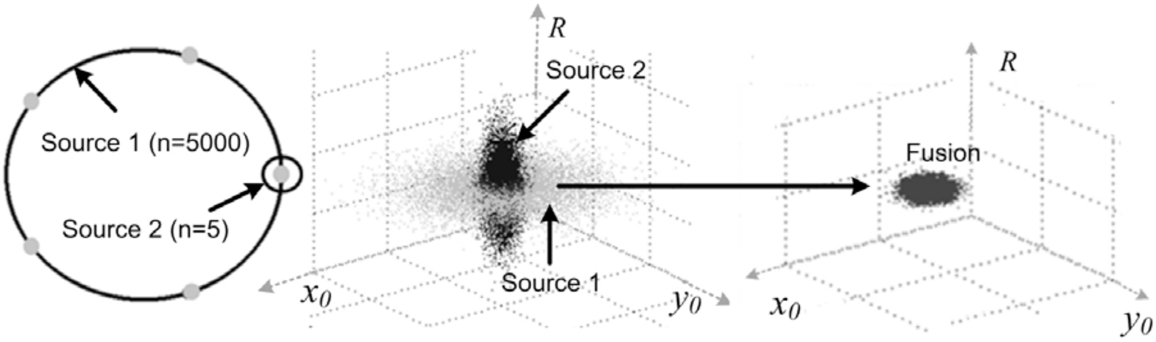

Fig. 8. Simulation of data fusion of circle features.

5 Stitching of point clouds

Stitching, as the process of combining multiple point clouds, is a typical problem of data fusion for data collecting. The application of stitching is required in large-scale and multi-scale (multi-resolution) metrology, where providing high spatial resolution over a large field of view (range-resolution problem) is required. Also, stitching procedures for complex measurement objects are to be applied when a sequence of measurements is necessary to cover the object surface. An example of the stitching of the form measurement of rotationally symmetric workpieces is discussed in this paper.

5.1 Measurement task and realization

Workpieces with a rotational symmetry are usually measured with form testing instruments. As an alternative we demonstrate the application of optical 3D measurements, using a stitching algorithm.

For our experiments we applied an ATOS measurement system (GOM company). It consists of two measuring cameras with overlapping fields of view and one projector, and combines the photogrammetric principle and the fringe projection method (Fig. 9a). The measurement object has to be fixed horizontally to allow stable positioning during the measuring and access to the surface. For this purpose, special calibrated supports are used. The support (Fig. 9b) was calibrated with a CMM. The proposed approach can be used for macro-shape as well as for micro-shape with the following application of the measurement results (inspection, etc.).

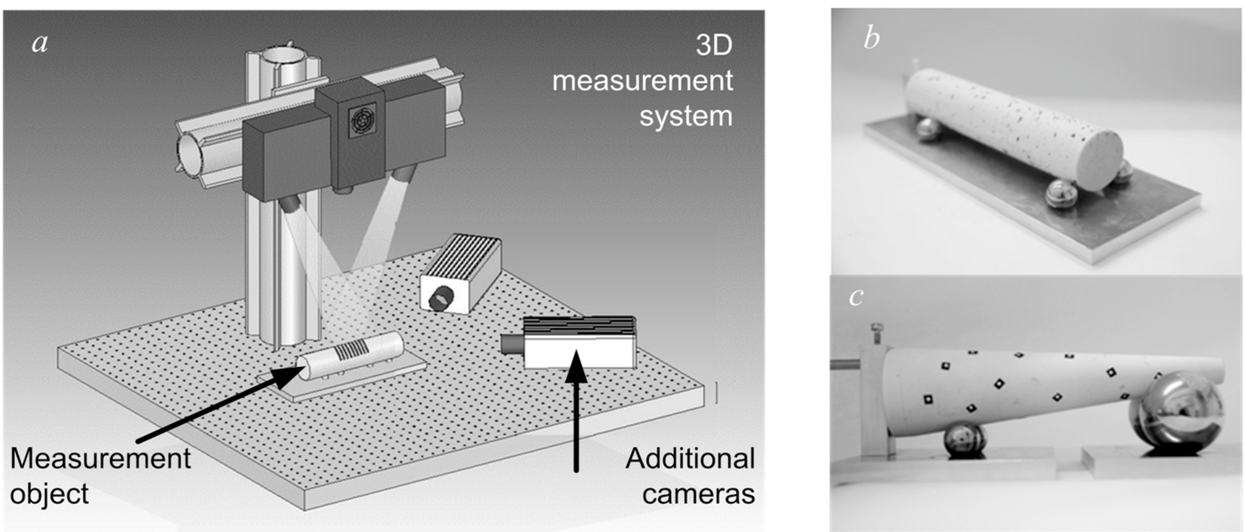

Fig. 9. Measurement system (a) and experimental supports for measurements of rotationally symmetric workpieces (b, c).

A single measurement yields one patch (Fig. 10), which consists of a large number of points in a limited area. It is possible to evaluate the geometrical parameters of the rotationally symmetric object (cylinder). The influence of the sampling strategy on the measurement uncertainty of the features is studied in [15].

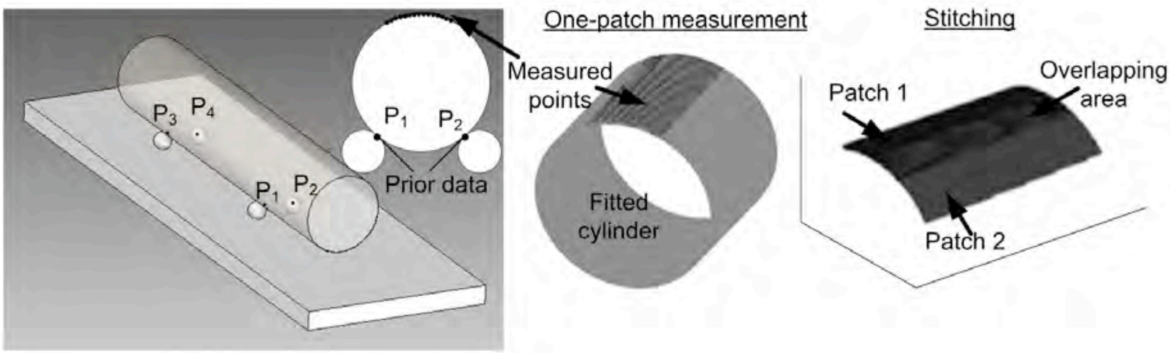

Fig. 10. Form measurement of cylindrical workpieces.

Modification of fitting algorithm by the addition of the data of support calibration allows the improving of the fitting results. We consider this as the kind of data fusion for the data improving, where the support is a *complementary data source* to the measurement data.

For collecting information about form deviation, a number of patches are needed to cover the surface of the object. The rotation of the object is performed manually.

5.2 Point cloud processing

Processing of point clouds includes outlier removing, trimming, filtering and transformation to a certain format if necessary. A point cloud, obtained from measurement, represents scattered data (points have no structure or order between their relative locations).

For some algorithms of the point clouds' processing, a regular grid for data is required. For example: filtering, point-to-point matching or multi-resolution fusion. Regularization of original point clouds requires interpolating scattered data. Commonly Delaunay triangulation is used for interpolation. For rotationally symmetric objects, the processing in cylindrical coordinates (ρ, θ, z) can be used [9]. The advantage of such an approach is the univocal dependence between coordinates:

$$\rho = \sqrt{x^2 + y^2}, \theta = \arcsin(y/\rho) = \arccos(x/\rho).$$

Representation in cylindrical coordinate system enables the application of different algorithms to interpolate data (for example in Matlab). In this work, linear interpolation has been applied to get a regular grid for cylindrical coordinates. The problem of the measurement uncertainty propagation in a case of *interpolation* is analyzed in [16]. Regularized data can be filtered using areal filters. Different classes of filters are proposed in ISO/TS 16610 series of standards: linear, morphological, robust and segmentation filters. Detailed analysis of the multi-dimensional filtering is given in [17]. The problem of uncertainty propagation in the case of *filtering* (of profile measurements) is investigated in [18]. An example of the processing of a point cloud (representing one patch) is given in Fig. 11.

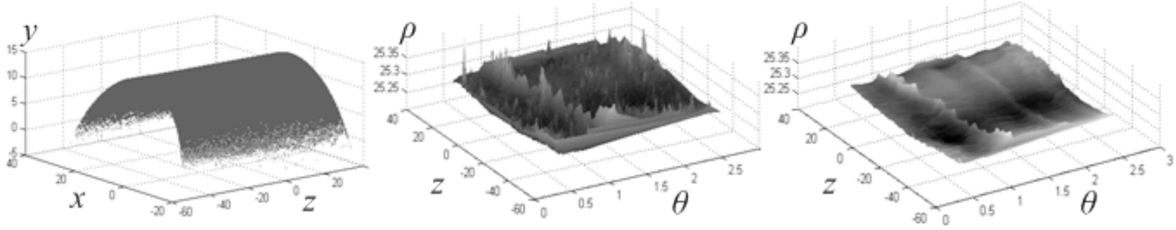

Fig. 11. Point cloud processing: (a) original point cloud (one patch); (b) regularized data in cylindrical coordinate system; (c) filtered data.

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For the least squares reference cylinder (LSCY) estimation, the following optimization problem to be solved is:

$$\min\left[(x-(k_x\cdot z+a))^2+(y-(k_y\cdot z+b))^2-R^2\right], \quad (3)$$

where k_x, k_y - tilt parameters; a, b - translation parameters; R - radius of the cylinder.

The expression (3) can be modified by adding the prior data from the calibrated supports and by weighting the components of the optimization problem. The weights are inversely proportional to the uncertainty of the measured coordinates (see (2)). Uncertainty associated with fitted parameters is discussed in [15, 19]. As well, the other kinds of reference cylinder estimation (associated cylinder fitting) are applied: minimum zone (MZCY), minimum circumscribed (MCCY), maximum inscribed (MICY).

5.3 Registration

Performing multiple measurements (multi-view measurements) requires the appropriate processing of multiple point clouds. First, the multiple point clouds have to be transformed to common coordinate system. Three different approaches of registration were implemented and analyzed. The *first approach* involves the use of an additional measurement channel for the transformation parameters' measurement. We used two cameras to estimate the parameters of 3D transformation by the photogrammetric principle and created a random pattern on the base of the cylinder (Fig. 12a). This requires the evaluation of the reference points' coordinates on the random pattern before and after transformation (cylinder rotation). The transformation matrix is evaluated based on these coordinate pairs. These operations are carried out by ARAMIS software (GOM company). This approach is effective with respect to the costs of the additional measurement channel and its input to the total uncertainty.

The *second approach* for stitching requires using circular markers (Fig. 9c, Fig. 12b). In this case the ATOS software can be used, but there are some challenges: the choice of the diameter and the number of circular markers. A deformed circle on the cylindrical surface changes the form of its projections on the images taken by cameras, which in turn induces an offset of the center of the ellipse and causes a *systematic error*. Obviously, one needs to reduce the diameter of the circular marker, but the minimal diameter value is limited by the resolution of the cameras. At least three markers in each overlap are needed. To reduce the uncertainty, one can increase the number of markers (Section 3.2, [10]).

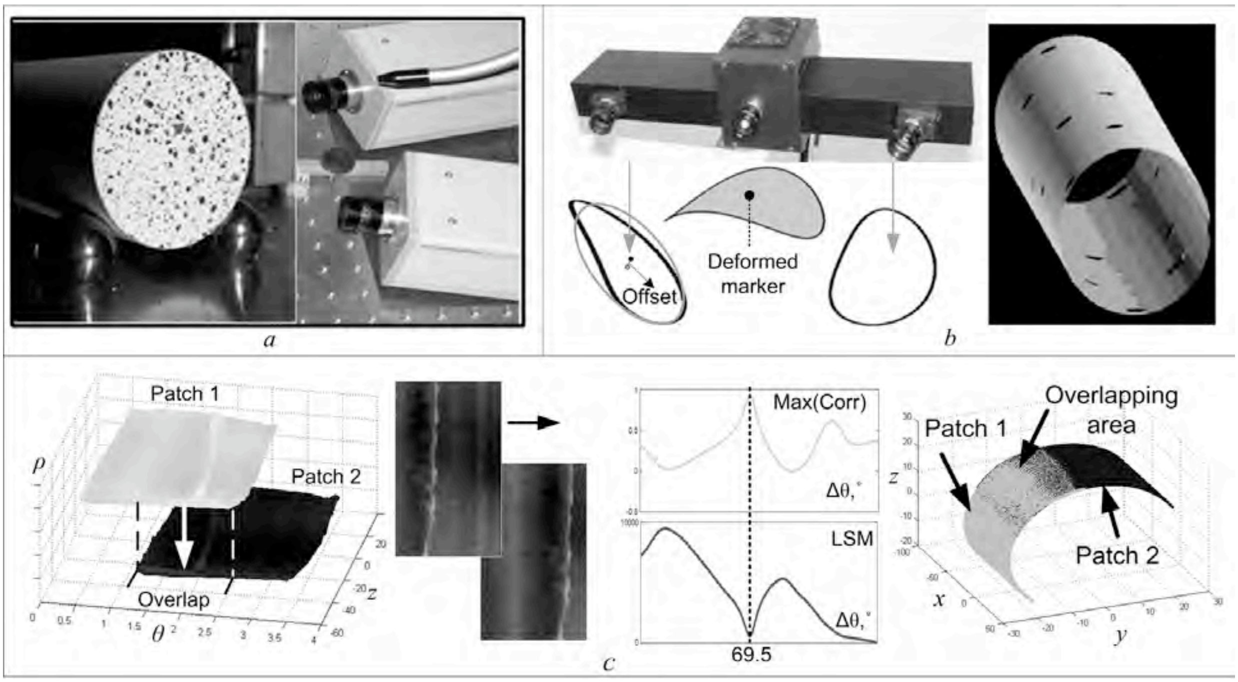

Fig. 12. Registration approaches: (a) additional measurements using two measuring cameras; (b) based on circular markers and LSM (ATOS software); (c) based on surface non-idealities and point-to-point matching (LSM or cross-correlation method).

The *third approach* involves the non-idealities of the surface in the overlapping zone. In Fig. 12c the result of matching two filtered patches of the cylinder is shown (patches are separated on the ρ-axis for demonstration). Rotation of the cylinder in Cartesian coordinate system is identical to the translation in cylindrical coordinates. Two criteria for the point-to-point matching have been tested: maximum of the cross-correlation function and minimum of the sum of distances. The results obtained from both criteria show good agreement (Fig. 12c). In the given example the accuracy of the stitching is limited by the discretization step for the obtained regular grid. The possibilities of stitching with sub-pixel accuracy are discussed in [20].

After the transformation parameters are evaluated, point clouds have to be transformed. The challenge is dealing with points in overlapping areas, where the "double" point clouds are obtained.

5.4 Measurement uncertainty analysis and reduction

In Fig. 13 a stitching process simulation is presented. The simulation is carried out with the circular object: the segments of the circle (patches) have to be stitched to form the part of the circular profile. Both segments contain four reference points that are used for the transformation parameters' calculation. The reference points belong to the overlapping area (20%).The number of points and the size of overlap can be easily modified in the Matlab script used for the simulation. For each point, the uncertainty is simulated as the a random non-correlated process with Gaussian distribution (standard uncertainty: 0.01 mm). The uncertainty of the reference points is simulated in the same way, but it is supposed that their coordinates are more precise (standard uncertainty: 0.001 mm). The transformation parameters are estimated by LSM for each iteration of the Monte Carlo simulation.

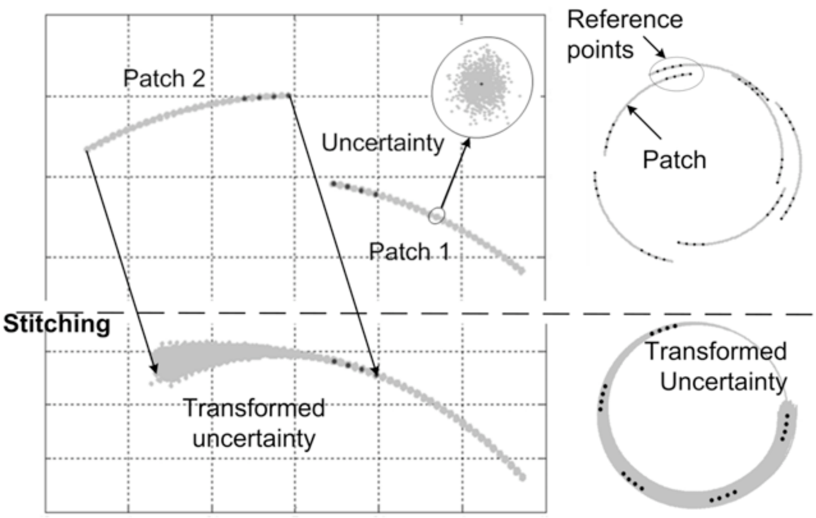

Fig. 13. Stitching process simulation. Left: Input data for simulation of two patches. Four reference points in 20% overlap, standard uncertainty of the reference points' coordinates 0.001 mm, standard uncertainty of the coordinates of the rest of the points in point clouds 0.01 mm. Right: Uncertainty cumulation in sequential stitching for the circle (R=10 mm).

Due to the uncertainty of the transformation parameters, the uncertainty of the points' coordinates in the transformed patch increases depending on the location of the point with respect to the overlap. For the sequential stitching, this can be used for the weighting of the coordinates of points in overlapping zones. To avoid uncertainty cumulating (Fig. 13, right), the following solutions are offered:

- simultaneous optimization for all measured patches (for spherical measurements it is used in [21]);
- use of additional reference points for the construction of the reference coordinate system in transformation process (indirect evaluation of the transformation parameters with respect to the fixed reference points).

Fig. 14 shows the realization of such approach, where the multilateration method is used (mentioned in Section 2).

Simulations have shown that improvement of the stitching process is achieved by the minimization of the transformation parameters' uncertainty. This can be provided in the stage of the measurement process planning (size of overlap, number of reference points) and correct data processing (preprocessing, realization of LSM).

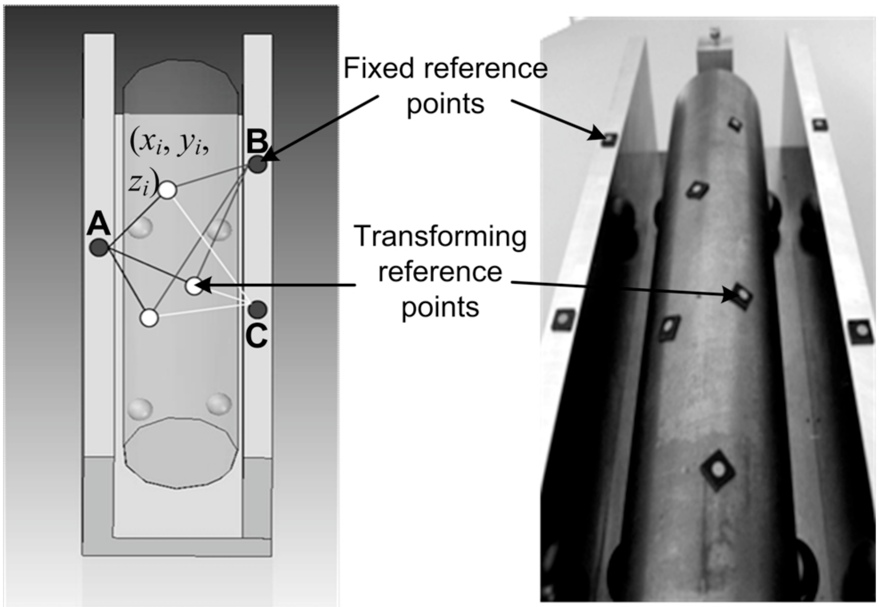

Fig. 14. Transformation parameters' estimation with use of the modified support. Fixed reference points create the local coordinate system of the support. The "fixed" markers **A**, **B**, **C** enable a realization of the trilateration method.

Conclusions

In coordinate metrology, data fusion is either for data improvement or data collection. The reliability and quality of the final measurement result depend not only on the single measurement quality characteristics, but also on the data fusion procedure realization, including the registration process. Registration includes coordinate transformation that leads to the measurement uncertainty transformation.

Data fusion requires many operations and represents a complicated multi-stage model. The structure of measurement uncertainty in a point cloud has to be taken into account. Since analytical derivations are complicated in this case, preferably the Monte-Carlo method is used for the uncertainty evaluation.

The improvement of the quality of the measurement result by data fusion is achieved by redundancy and appropriate weighting techniques.

An example of the form measurement of rotationally symmetric workpieces was discussed. The multi-view measurement strategy requires data fusion for data collection. The measurement uncertainty reduction is achieved by the complementary data source (calibration data of support) and the stitching procedure improvement.

Acknowledgements

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