Simultaneous measurement of resistance and temperature changes in bridge circuits

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Abstract

Wheatstone Bridge Circuits are commonly used in sensor applications to measure the resistance change of sensor elements, which represent typical metrics such as pressure, force or magnetic flux density. In many applications, for example, magnetoresistive or pressure-sensitive sensor elements are used in Wheatstone bridges. A temperature change of the Wheatstone bridge leads not only to a change of the resistance of the sensor elements, but also of the reference resistors. The reason for this is the (mostly positive) temperature coefficient of the resistors and also the dependence of the sensor effect on the temperature. In order to obtain a more accurate result, the temperature of the bridge circuit should therefore be known. By using several different voltages in a bridge circuit and knowing the temperature coefficients, it is possible to determine the exact resistance change of the sensor elements and the temperature change simultaneously without using an additional temperature sensor. A possible offset of the bridge circuit due to a mismatch of the four resistors is not relevant.

Keywords: Bridge-Circuit, Temperature, resistance change

1. Method for determining the resistances in a Wheatstone bridge under reference conditions

In this proposal it is essential to know the exact values of the four resistances in the Wheatstone bridge under reference conditions, i.e. the temperature is given; then the temperature change is zero (dT = 0, therefore the temperature coefficients a and b are not relevant) and there is no change in resistance as no measured quantity is applied (then: dR = 0).

To accurately measure each of the four resistors, a floating zero-resistance ammeter is suggested, as shown in Figure 1a and 1b. It is important to note that the bridge circuit with R1 to R4 has already been produced and is ready for use. The basic idea is to short-circuit one of the two resistors connected in series connection (in Figure 1a and 1b: node 1 and node 2 are short-circuiting so that R4 is no longer relevant) and measure the current (here: I(R3)) using operational amplifiers so that, for example, the resistance of R3 can be determined by the voltage U3 (then U3 = U2) divided by the measured current I(R3); here: I(R3) = I(R3) with I(R3) as the current through the (reference) resistor R3 in the Wheatstone bridge; all this is resulting in:

\[ R_3 = \frac{U_3}{I(R_3)} = \frac{U_2}{I(R_3)} \]

The current I(R3) (= I(R3)) can be derived by using the difference of the voltages at the output of the two operational amplifiers (UA1 - UA2) according to equation (1); it is important to know that (R5 - R7) is equal to (R6 - R8).

Equation (1) for I(R5) results in:

\[ I(R_5) = \frac{U_{16} - U_{12}}{R_5 \left( 1 + \frac{R_4}{R_6} \right)} = I(R_5) = I(R_5) \] (1)

![Fig. 1a: Floating zero-resistance ammeter for measuring the resistances at reference conditions (dT = 0 and dR = 0) (for further details see [1])](image)

In this example k3R0 is the value of the resistance at dT = 0: R3 = Uv/I(R3) = 1.05 kΩ which corresponds with the correction factor k3 = 1.05 multiplied with the nominal resistance R0 = 1 kΩ. All this can be realised at the end of the production process and possibly the data can be stored in the sensor (e.g. by means of laser fuses) and can be used in signal conditioning circuits. With a micro-
controller the change of the resistance dR and the change of the temperature dT can be evaluated as described in the following chapters.

![Diagram](image)

**Fig. 1b:** Floating zero-resistance ammeter for measuring the resistances at reference conditions (dR = 0 and dT = 0) (for further details see [1])

2. Method for simultaneously determining the change in resistances and temperature in Wheatstone bridges supplied with a voltage source

A new concept for the simultaneous determination of the change in resistance dR of sensor elements and the change in temperature dT in Wheatstone bridges is proposed. The basic idea is to consider the known temperature coefficients of all resistors used in the bridge circuit. Then the absolute change of the resistance dR and the temperature dT compared to the reference conditions (with dR = 0 and dT = 0) can be determined by using different voltages of the bridge circuit. The equations depend on the circuit, so a quarter bridge with one sensor element, a half bridge with two sensor elements or a full bridge circuit with four sensor elements have different equations. For each sensor element, the relative change of the resistance dR/R0 is the same when the same measured variable is applied, where R0 represents the nominal resistance at reference conditions.

2.1 Quarter bridge circuit with one sensor element and voltage source

In the quarter bridge depicted in Figure 2, the sensor element is represented by the resistor R1; R0 is the nominal value (here: 1 kΩ) and k1 is the factor that determines the exact value of R1 at reference conditions – same is valid for k2 for R2, k3 for R3 and k4 for R4; b is the constant temperature coefficient of R1 and dR represents the absolute change of R1 due to the measured variable. The resistors R2, R3 and R4 are reference resistors with different constant temperature coefficients (TC) a and g with a ≠ g. It is possible: a = b or g = b with b as TC of the sensor element; this means that either R3 or (R2 and R4) can be replaced by an insensitive sensor element which only reacts to temperature changes but not to changes in the measured variable. But always the temperature coefficients a and g of R3 and R4 need to be different: a ≠ g.

![Diagram](image)

**Fig. 2:** Quarter bridge circuit with one sensor element (R1) and three reference resistors (R2, R3, R4)

**Sensor resistance:**

\[ R_s = k_1 \cdot R_0 \cdot (1 + b \cdot dT) + k_1 \cdot dR \]

**Reference resistances:**

\[ R_2 = k_2 \cdot R_0 \cdot (1 + g \cdot dT) \]

\[ R_3 = k_3 \cdot R_0 \cdot (1 + a \cdot dT) \]

\[ R_4 = k_4 \cdot R_0 \cdot (1 + g \cdot dT) \]

In equation (2) the calculation of the temperature change dT is shown; equation (3) describes how the relative change of the resistance dR/R0 of the sensor element can be derived with R0 as nominal resistance. Figure 3 shows the result of this calculation in a simulation with LTSpice [3]. It can be seen that both the temperature change dT and the relative resistance change dR/R0 can be derived simultaneously using equations (2) and (3).

For the change of the temperature dT equation (2) is used:

\[ dT = \frac{k_3 \cdot U_y - U_3 \cdot (k_2 + k_4)}{U_3 \cdot (a \cdot k_3 + g \cdot k_4) - a \cdot k_1 \cdot U_y} \quad (2) \]

and equation (3) for the relative change dR/R0 of the resistance of the sensor element R1:
\[
dR = \frac{k_3 \cdot U_v - U_i \cdot (k_3 + k_4)}{k_1} \\
\frac{U_i - U_i - U_i}{U_i - U_i - U_i} \\
\frac{k_2 \cdot g \cdot U_i - k_2 \cdot b \cdot (U_i - U_i)}{(U_i - U_i)} \\
\frac{k_1 \cdot (a \cdot k_1 + g \cdot k_1) - a \cdot k_1 \cdot U_i}{U_i - U_i - U_i} \\
\frac{k_1 \cdot (U_i - U_i - U_i)}{U_i - U_i - U_i}.
\]

(3)

\[
dR = \frac{k_3 \cdot U_v - U_i \cdot (1 + a \cdot dT) - k_4 \cdot U_i}{k_4 \cdot U_i}
\]

(4)

\[
dT = \frac{k_3 \cdot U_v - k_1 \cdot (U_i - U_i)}{k_3 \cdot (U_i - U_i) - k_4 \cdot b \cdot U_i} + k_1 \cdot a \cdot (U_i - U_i) - k_4 \cdot b \cdot U_i + k_1 \cdot (U_i - U_i) - k_4 \cdot U_i + k_4 \cdot U_i \cdot k_1 \cdot b \cdot (U_i - U_i) - k_4 \cdot U_i.
\]

(5)

The results of equations (4) and (5) are depicted in Figure 5; again, the change in temperature dT and the change in resistance dR of the sensor element can be calculated using equation (4); then the change in resistance dR can be derived using equation (5).

2.2 Half bridge circuit with two sensor elements and voltage source

In the half bridge shown in Figure 4 two sensor elements R1 and R4 are used. The resistors R2 and R3 represent the reference resistors with different temperature coefficients (TC) a and g: a ≠ g.

![Half bridge circuit with two sensor elements and voltage source](image)

Fig. 4: Half bridge circuit with two sensor elements (R1, R4) and two reference resistors (R2, R3)

Similar to the previous chapter, dR (or dR/R0) and dT can be calculated using equations (4) and (5). In this example the following also applies: g = b; so R2 can be realised as an insensitive sensor element. It is important that the TC of the reference resistors R2 and R3 are different: a ≠ g.

Equation (4):

\[
dT = \frac{k_3 \cdot U_v - k_1 \cdot (U_i - U_i) + k_1 \cdot a \cdot (U_i - U_i) - k_4 \cdot b \cdot U_i + k_1 \cdot (U_i - U_i) - k_4 \cdot U_i + k_4 \cdot U_i}{k_4 \cdot U_i}.
\]

Equation (5):

3. Method for simultaneously determining the resistance and temperature change of Wheatstone bridges with additional resistor and voltage source

If there is an additional resistor R5 used that is connected in series to the bridge circuit (see Figure 6), it is also feasible to calculate the change of the temperature dT and the change of the resistance dR of the sensor elements. The idea is to get the information about the
total current flowing through the bridge circuit in order to calculate $dT$ and $dR$. This is shown in the following chapters for quarter bridge, half bridge and full bridge Wheatstone circuit.

### 3.1 Quarter bridge circuit with one sensor element and additional series resistor

Figure 6 shows a quarter bridge circuit (with one sensor element $R_1$ and three reference resistors $R_2$, $R_3$, $R_4$) with an additional resistor $R_5$. As in the previous chapters, it is possible to derive the temperature change $dT$ and the resistance change $dR$ (or relative change $dR/R_0$) of the sensor element by using voltages, temperature coefficients (TC) and resistances at reference conditions - see equations (6) and (7); the temperature change $dT$ as a result of equation (6) is used in equation (7). Here the temperature coefficients $a$ of the three reference resistors may be similar (without $R_5$ they need to be different, see chapter 2.1) – the temperature coefficient $a$ can be similar to the temperature coefficient $b$ of the sensor element $R_1$; therefore: $a = b$. If $R_5$ is not constant, then the temperature coefficient $f$ (with $R_5 = k^5 \cdot R_0 \cdot (1 + a \cdot dT)$, see Fig. 6: $f \neq a$) and the temperature of $R_5$ must be known; it is still possible: $a = b$. $R_5$ may be located in close proximity to the signal conditioning circuit (e.g. microcontroller or electronic control unit); the temperature change and hence the change of $R_5$ is then usually limited.

$$dT = \frac{k_3 \cdot U_2 + U_3}{a \cdot (U_i - U_{br})} \cdot \frac{k_5 \cdot R_0}{R_5} - \frac{1}{a} \quad (6)$$

with $R_5$

as constant resistor and equation (7):

$$dR = \frac{k_2}{k_1} \cdot (1 + a \cdot dT) \cdot \left( \frac{U_i}{U_{br} - U_i} \right) - (1 + b \cdot dT)$$

In Figure 7 the result of the simulation with LTspice [3] is shown – again the change of the temperature $dT$ and the change of the resistance $dR$ of the sensor element can be calculated by using different voltages and temperature coefficients.

If the voltage source $V_1$ and the resistor $R_5$ are substituted by a current source $I_5$, equation (6) for changing the temperature $dT$ is changed to equation (8); for $dR$ equation (7) can be used.

$$dT = \frac{k_3 \cdot U_2 + U_3}{a \cdot I_5 \cdot k_3 \cdot R_0} - \frac{1}{a} \quad (8)$$

![Fig. 6: Quarter bridge circuit with additional resistor $R_5$ and a bridge circuit with one sensor element ($R_1$) and three reference resistors ($R_2$, $R_3$, $R_4$)

![Fig. 7: Quarter bridge circuit (see Fig. 6) depending on change of $R$ ($dR$) with temperature change $dT$ as parameter: Top: Temperature change $dT\ Bottom: Resistance change $dR$](image)

### 3.2 Half bridge circuit with two sensor elements and additional series resistor

Figure 8 shows a half bridge circuit with two sensor elements ($R_1$ and $R_4$), two reference resistors ($R_2$ and $R_3$) and additionally the resistor $R_5$ in series connection to the bridge circuit. The temperature coefficient $f$ of the series resistor $R_5$ must differ from the temperature coefficient $a$ of $R_2$ and $R_3$ (hence: $f \neq a$), but it is possible: $a = b$ with $b$ as temperature coefficient of $R_1$ and $R_4$.  

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Equation (9a) applies to the change in temperature $dT$ under the condition that the value of R5 is constant (possible: $a = b$) and equation (9b) applies to $dT$ if R5 (with $f$ as constant temperature coefficient) is at the same temperature as the bridge circuit; $dT$ can be used to calculate $dR$ (or $dR/R_0$) in equation (10).

$$dT = \frac{k_1 \cdot U_2 + U_3}{a \cdot (U_y - U_{BR}) \cdot k_1 \cdot R_0 - R_5} - \frac{1}{a} \quad (9a)$$

R5 as constant resistor and equation (9b) with $f$ as TC of R5:

$$dR = \frac{k_2 \cdot k_3 \cdot U_y}{U_{BR} \cdot k_2} \cdot \frac{(U_{BR} - U_{BR})}{U_2 \cdot (k_1 - k_2)} - \frac{-k_2 \cdot k_3 \cdot a}{U_2 \cdot k_4} \cdot f \quad (9b)$$

$$\begin{align*}
\frac{dR}{R_0} &= \frac{U_{BR} \cdot k_3 \cdot U_2}{U_1 \cdot k_4} \\
&+ \frac{dT \cdot [U_{BR} \cdot k_2 \cdot a - U_1 \cdot (a \cdot k_3 + b \cdot k_4)]}{U_1 \cdot k_4} \quad (10)
\end{align*}$$

In Figure 9 the results of equation (9b) and (10) are depicted. The temperature change $dT$ and the change of the sensor resistance $dR$ can be determined again simultaneously. Similar to the previous chapter, the voltage source V1 and the resistor R5 can be replaced by a current source I5; then equation (8) is valid again.

Fig. 8: Half bridge circuit with additional resistor R5 and a bridge circuit with two sensor elements (R1, R4) and two reference resistors (R2, R3)

3.3 Full bridge circuit with four sensor elements and additional series resistor

A full bridge circuit consists of four sensor elements as shown in Figure 10. Then the two sensor elements R1 and R4 must show reciprocal behaviour compared to the other two sensor elements R2 and R3, so that the resistance of the bridge circuit can be assumed to be constant - only depending on temperature change $dT$. Equation (11a) applies to the change in temperature $dT$ provided that the value of R5 is constant; equation (11b) applies to $dT$ when R5 is $[R_5 = k_5 \cdot R_0 \cdot (1 + f \cdot dT)]$, where $f$ is the constant temperature coefficient of R5 at the same temperature as the bridge circuit; $f$ must be different to b as TC of the sensor elements ($f \neq b$); equation (12) can be used to calculate $dR$ or the relative change $dR/R_0$.

Fig. 9: Half bridge circuit (see Fig. 8) depending on change of R ($dR$) with temperature change $dT$ as parameter: Top: Temperature change $dT$ Bottom: Resistance change $dR$

Fig. 10: Full bridge circuit with four sensor elements (R1, R2, R3, R4) and an additional series resistor R5
Equation (11a) with R5 as constant results in

\[
d T = \frac{U_{ab} \cdot (k_1 + k_2 + k_3 + k_4) \cdot R_5}{b - (U_v - U_{ab}) \cdot R_5 \cdot (k_1 + k_2) \cdot (k_3 + k_4)} - \frac{1}{b}
\]

and equation (11b) with f as TC of R5

\[
d T = \left( U_v - U_{ab} \right) \cdot (k_1 + k_2) \cdot (k_3 + k_4) + \frac{U_{ab} \cdot (k_1 + k_2 + k_3 + k_4) \cdot f + b \cdot (U_v - U_{ab}) \cdot (k_1 + k_2) \cdot (k_3 + k_4)}{1 + b \cdot d T}
\]

\[
\frac{d R}{R_0} = \left[ \frac{U_{ab} \cdot k_1 - U_1 \cdot (k_1 + k_2)}{U_1 \cdot (k_1 - k_2) - U_{ab} \cdot k_1} \right] \cdot (1 + b \cdot d T)
\]

\[d T = 100 \text{ K}\]

\[d T = 50 \text{ K}\]

\[d T = 0 \text{ K}\]

\[d R / \Omega\]

\[d R / \Omega\]

\[d R / \Omega\]

\[d T = 100 \text{ K}\]

\[d T = 50 \text{ K}\]

\[d T = 0 \text{ K}\]

4.1 Quarter bridge circuit using one AMR sensor element

In Figure 12 a quarter bridge circuit with one AMR sensor element R1 is shown. The TC of R3 and R4 must be different (a ≠ g); a as TC of R3 may be equal to b as TC of the AMR sensor element (a = b), e.g. by using an insensitive AMR-sensor for R3. The change in temperature dT can be derived using equation (13) which is similar to equation (2). As depicted in Figure 13, the typical AMR characteristic (R depending on the magnetic field strength H in the direction of the hard axis) is influenced by the temperature change dT: The higher dT, the lower the change of the resistance dR due to the negative TC (-0.002/K) of the AMR effect.

\[
d T = \frac{k_1 \cdot U_v - U_3 \cdot (k_1 + k_4)}{U_3 \cdot (a \cdot k_3 + g \cdot k_4) - a \cdot k_3 \cdot U_v}
\]
4.2 Full bridge circuit using four AMR sensor elements

In Figure 14 a full bridge circuit with four AMR-sensor elements R1 to R4 is depicted. The resistors R1 and R4 must exhibit opposite behaviour to R2 and R3, which can be achieved by barber pole structures. In Figure 15 the results of the simulation is depicted: The temperature change $dT$ (see equation (14)), the resistance change $dR$ and also the output voltage $U_0$ can be determined simultaneously using the corresponding equations.

Equation (14) is valid for the full bridge circuit under the condition that $R_5$ is constant (see equation (11a)):

$$dT = \frac{U_{BR} \cdot (k_1 + k_2 + k_3 + k_4) \cdot R_5}{b \cdot (U_V - U_{BR}) \cdot R_5 \cdot (k_1 + k_2) \cdot (k_3 + k_4)} - \frac{1}{b}$$

![Fig. 14: Full bridge circuit with four sensor elements (AMR-sensors R1-R4)](image)

5. Conclusion

A new concept is described which allows the simultaneous measurement of the change in resistance $dR$ and the change in temperature $dT$. This is achieved by using the applied voltages and temperature coefficients of all resistors in the Wheatstone bridge. This concept is applicable to quarter, half and full bridge circuits with different numbers of sensor elements. It is shown how the relevant information (resistors R1 to R4) is determined in an already manufactured Wheatstone bridge circuit with an operational amplifier circuit. A possible offset of the bridge circuit due to a mismatch of the four resistors is not relevant. The additional information about the temperature can help to obtain more accurate results. The bridge circuit can not only be used for compensation of the temperature influence, but also as a temperature sensor for condition monitoring or diagnostic functions in the application. Thus, the bridge circuit can provide two different information simultaneously: resistance change $dR$ and the temperature change $dT$. In this example, the temperature coefficient is assumed to be constant - if this is not the case, the calculation can be performed using a numerical approach. A patent application has already been filed.

References