# Heterodyne “weak measurements” of nanorad beam deflections

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## 1. Introduction

Generally a measurement yields (eigen)values that can be distinguished and recorded by a suitable measuring device. Aharanov, Albert, and Vaidman (AAV) considered the case of “weak measurements” in quantum mechanics, where the eigenvalue-spectrum is not properly resolved [1]. AAV showed that if a system is prepared such that prior to the measurement it is in a well-defined state, then it is helpful to choose a suitable post-selection state, which should be almost orthogonal to the initial state, and which is able to give rise to arbitrarily large expectation values.
Here, we discuss how weak value amplification can be adapted to measure the optical activity of chiral liquids. The hallmark of optical activity is that the liquid has different refractive indices for right- and left-circularly polarized light, which causes the rotation of the plane of polarization of a linearly polarized light beam traversing a chiral liquid. The small difference in refractive indices (<< $10^{-7}$) found in naturally optically systems can be detected in transmission (polarimetry). However, as Fresnel first discussed, it may also be detected in refraction. A linearly polarized light beam incident at an interface between a chiral and an achiral medium will split into its two circular polarization components, as the two components refract with different angles of refraction. A position sensitive detector may be used to register the difference in beam positions [4]. For chemical and pharmaceutical applications it is of interest to measure small optical activities (with correspondingly small refractive index differences) in minute liquid samples. In order to increase the sensitivity of the measurement it becomes necessary to increase the separation between the two split circular beam components.
We show that conventional weak value amplification, as reported in the literature before, does not provide sufficient information to determine the handedness of a chiral liquid. Instead we describe how weak value amplification combined with a heterodyne detection scheme can measure both the concentration and the handedness of the solution. The (magneto-optical) Faraday effect in a glass prism is used as a model system for optical activity. The linear polarized beam is split into its circular polarization components at the interface.. To amplify these small angular displacements (typically a few nanoradians) of the two circular beam components we have implemented a weak measurement with a suitable post-selection scheme. It is shown, that the signal is proportional to the circular birefringence, and that it is sensitive to the sign of the optical activity.

## 2. Splitting up circular polarized beam components via optical activity

Any isotropic medium becomes optically active and uniaxial in the presence of a longitudinal magnetic field, which is also known as the Faraday effect. We consider the Faraday effect in a glass prism as a model system for an optically active solution with an angled interface. The plane of polarization of a linearly polarized electromagnetic wave at wavelength $\lambda$ is rotated by an angle $\alpha$ as the wave propagates a distance l inside the medium along the direction of the field, which is

$$\alpha = \frac{\pi l}{\lambda}(n^+ - n^-) = VBl$$

where V is the frequency dependent Verdet constant and B is the magnetic field strength. When refracted at a boundary formed by a optical active medium and a isotropic medium left- and right-circularly polarized waves or wave components must independently obey Snell’s law. The two circularly polarized components are refracted into slightly different angles of refraction $\theta^+$ and $\theta^-$. The angular divergence, $\Delta\theta = \theta^+ - \theta^-$, between the two refracted circular polarization components is

$$\Delta\theta \approx \frac{\Delta n}{n_0}\frac{\sin\theta_i}{\cos\theta_a} = \frac{VB\lambda}{\pi n_0}\frac{\sin\theta_i}{\cos\theta_a}$$

where $\theta_a$ is the average of the two angles of refraction $\Delta n = n^+ - n^-$, and $n_0$ is the refractive index of the surrounding medium in which the wave is refracted.

## 3. Weak measurement of small beam deflections

### *3.1 The principle of weak measurments*

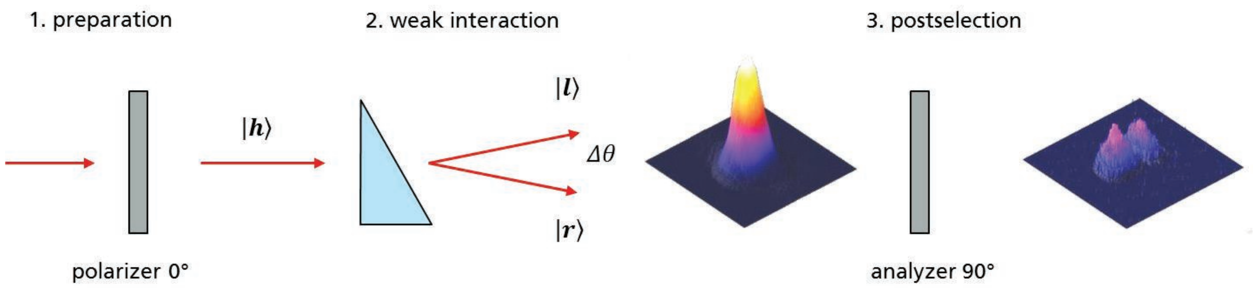


***Figure 1 Schematic of a weak measurement***

The principle of a weak measurement is based on three steps: 1) preparation of a state of the system (pre-selection), 2) weak interaction (perturbation) with the system, and finally 3) the post-selection of the final state (see Figure 1). A coherent light beam with a gaussian beam profile is linearly polarized at $\theta = 0°$ (pre-selection), so the electric-field vector can be described by

$$\boldsymbol{E_0}(x,y) = A_0(x,y)\, e^{i\,\boldsymbol{k_0}\cdot\boldsymbol{r}}|\boldsymbol{h}\rangle = E_0 exp\left[-\frac{x^2+y^2}{w^2}\right] e^{i\,\boldsymbol{k_0}\cdot\boldsymbol{r}}\begin{pmatrix}1\\0\end{pmatrix}$$

where *w* is the spot size of the beam. The beam is now split due to the weak interaction at the interface of the optical active medium. The left- and right-circularly polarized components propagate with a small angular divergence $\Delta\theta$ (see Sec. 2), which is small compared to *w*, and the electric-field vector a distance d after the interface can be written as

$$\boldsymbol{E}_1(x,y) = \frac{A_+}{2}\, e^{i\,\boldsymbol{k}_+\cdot\boldsymbol{r}}|\boldsymbol{l}\rangle + \frac{A_-}{2}\, e^{i\,\boldsymbol{k}_-\cdot\boldsymbol{r}}|\boldsymbol{r}\rangle = \frac{1}{2}\left[A_0(x+\Delta x,y)\, e^{i\,\boldsymbol{k}_+\cdot\boldsymbol{r}}\begin{pmatrix}1\\i\end{pmatrix} + A_0(x-\Delta x,y)\, e^{i\,\boldsymbol{k}_-\cdot\boldsymbol{r}}\begin{pmatrix}1\\-i\end{pmatrix}\right]$$

with

$$\Delta x = d\tan\left[\frac{\Delta\theta}{2}\right]$$

The vectors $|\boldsymbol{l}\rangle$ and $|\boldsymbol{r}\rangle$ are the Jones vectors for the left- and right-circularly polarized components, respectively. Because the angular beam shift is small compared to *w* both circular polarization components overlap to a large extent, except at the wings of the gaussian. It follows that the center of the resulting beam is still linearly polarized, and only the edges contain circular polarization components. Post-selection is achieved with an analyzer ($\theta_{pol} = 90°$) placed after the interface and in front of the detector. The center is extinguished and only the circular components at the edges of the gaussian pass the analyzer. This light intensity is directly proportional to the angular beam displacement $\Delta\theta$. The bigger $\Delta\theta$ the more intensity passes the polarizer. This can be seen from the field vector and the expression for the intensity after the analyzer

$$\boldsymbol{E_2}(x,y) = \frac{1}{2}\left[A_+\, e^{i\,\boldsymbol{k}_+\cdot\boldsymbol{r}}\, e^{i\,\theta_{pol}} + A_-\, e^{i\,\boldsymbol{k}_-\cdot\boldsymbol{r}} e^{-i\,\theta_{pol}}\right]|\boldsymbol{\theta_{pol}}\rangle \quad \text{with} \quad |\boldsymbol{\theta_{pol}}\rangle = \begin{pmatrix}\sin\,\theta_{pol}\\\cos\,\theta_{pol}\end{pmatrix}$$

$$I_2(x,y) = |\boldsymbol{E_2}(x,y)|^2 = \frac{1}{2}\left[A_+^2 + A_-^2 + 2A_+A_-\cos(\boldsymbol{\Delta k}\cdot\boldsymbol{r} + 2\theta_{pol})\right]$$

$\boldsymbol{\Delta k}$ is the difference between the two wavevectors corresponding to the separated circular polarized modes, and is given by:

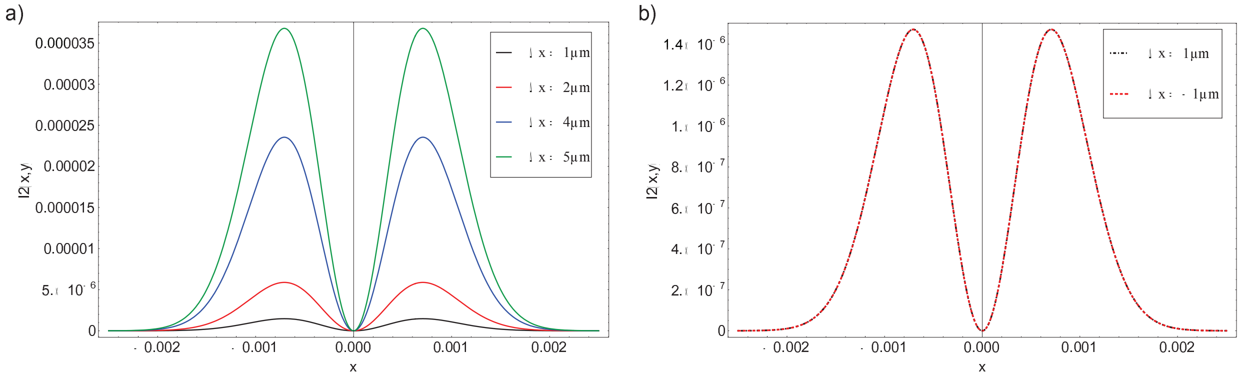


***Figure 2 Calculated Intensitydistribution I2(x,y) for a) different displacments Δx and b) for the same displacement but with oposite sign***

$$\boldsymbol{\Delta k} = \boldsymbol{k}_+ - \boldsymbol{k}_- = \frac{4\pi}{\lambda}\tan\left[\frac{\Delta\theta}{2}\right]\boldsymbol{\hat{x}}$$

Figure 2a is a plot of the intensitiy $I_3(x, y)$ calculated for different beam displacments Δx. Two peaks separated by a distance comparable to w are seen, where the peak intensity is proportional to the displacement of the beams. As can be seen in Fig. 2b, the peak intensity is not sensitive to the sign of $\Delta x$. It follows that the sign of the optical activity cannot be determined by this measurement. Therefore it becomes necessary to combine weak value amplification with a heterodyne detection scheme.

### *3.2 Weak amplified heterodyne measurments*

In order to extract the sign of the optical activity a heterodyne scheme is implemented. A Zeeman HeNe laser is used that emits two orthogonally circularly polarized modes with a frequency difference given by the Zeeman effect. A quarter-waveplate converts the circular components into vertical and horizontal polarization Gaussian modes which are each split into their circular polarization components at the chiral interface (vide supra). To amplify the very small angular displacement a post-selecting analyzer is introduced for one of the two beams before the heterodyne beat signal is observed at the detector. The direction of the laser beam is taken to be along the z-axis. Both modes have a 2D-gaussian beam profile with the same amplitude $E_0$ and a beam size *w*:

$$\boldsymbol{E_0}(x, y) = A_0(x, y)[\, e^{-i\omega_h t}\,|\boldsymbol{h}\rangle + \, e^{-i\omega_v t}\,|\boldsymbol{v}\rangle]\, e^{i\,\boldsymbol{k_0}\cdot\boldsymbol{r}}$$

Here $\omega_h$ and $\omega_v$ are the angular frequencies of the horizontally and vertically polarized modes, respectively, and $\boldsymbol{k_0}$ is the incident wavevector. The complex amplitudes for the horizontal and vertical polarized modes can be expressed in terms of the Jones vectors for left- and right-circular polarized light:

$$|\boldsymbol{h}\rangle = \frac{1}{2}[|\boldsymbol{l}\rangle + |\boldsymbol{r}\rangle]$$

$$|\boldsymbol{v}\rangle = \frac{1}{2i}[|\boldsymbol{l}\rangle - |\boldsymbol{r}\rangle] = \frac{1}{2}[i\,|\boldsymbol{r}\rangle - i|\boldsymbol{l}\rangle]$$

After refracting at the glass prism the circular polarized field components of both modes are independently refracted and separated. The right-circularly polarized components of the two laser modes are deflected in the same direction, similarly the left-components. After the post-selection analyzer one gets in a distance d behind the glass prism

$$I_3(x, y, t) = |\boldsymbol{E_3}(x, y)|^2 = \frac{1}{2}[A_+^2 + A_-^2 + (A_-^2 - A_+^2)\sin\Delta\omega t + 2A_+A_-\sin[\boldsymbol{\Delta k}\cdot\boldsymbol{r} + 2\theta pol]\cos\Delta\omega t]$$

The resulting intensity distribution has a contribution oscillating at the difference frequency $\Delta\omega$ of the two laser modes. The first term oscillating at $\sin\Delta\omega t$ is proportional to the difference of the two shifted amplitudes $A_+$ and $A_-$. It is expected to be sensitive to the sign of the lateral displacement $\Delta x$ (see Fig.3), and therefore sensitive to the sign of the optical activity.

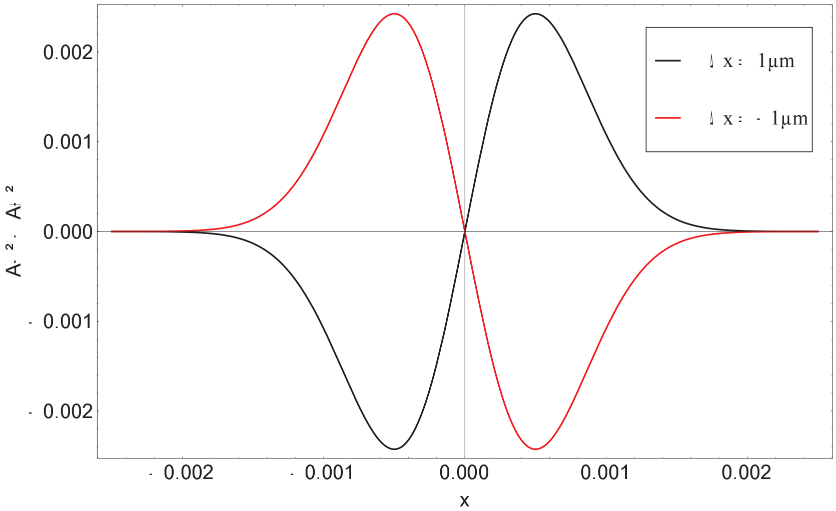


***Figure 3 Difference of the Amplitudes $A_+$ and $A_-$ for $\Delta x$ and $-\Delta x$***

## 4. Results

The setup shown in Figure 2 was used for all measurements. A HP 5517B Zeeman-HeNe-Laser was used as a light source, which emits both orthogonal linear polarized modes with an frequency difference of $\Delta\omega = 2.5\ MHz$ and a center wavelength of $\lambda$ = *633nm*. The light passes through the 30°-sample prism (SF11-glass) which is placed inside a homogenous longitudinal magnetic-field. The post-selection is done by a Glan-Thompson-polarizer before a position sensitive detector (PSD, UDT SPOT-9DMI.). The latter is placed 60 cm after the prism. The PSD is wired such that the left and right-side signals can be electronically subtracted and sent to a lock-In amplifier, which is locked to the difference frequency $\Delta\omega$.

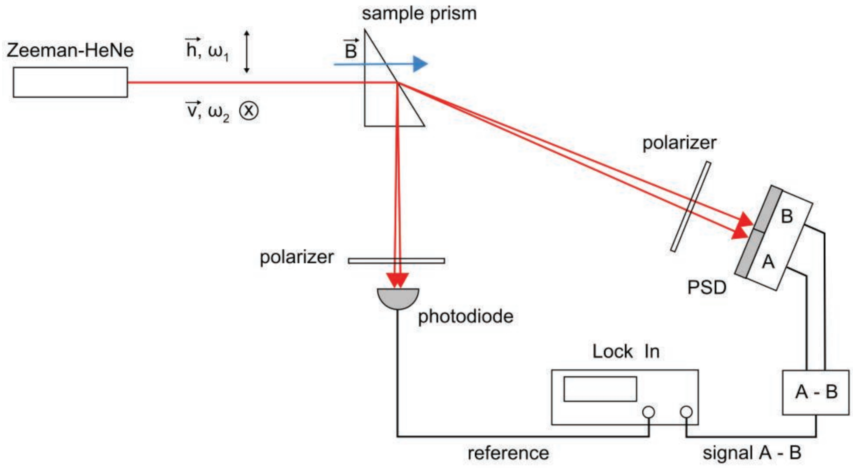


***Figure 4 Experimental setup used for the weak amplified heterodyne measurements.***

The in-phase component (denoted by *X(B)*) of the lock-in represents the term oscillating with $\cos \Delta\omega t$. The out-of-phase component (denoted by *Y(B)*) is the signal oscillating with $\sin \Delta\omega t$. At the beginning of each measurement the beam was centered on the PSD.

At first the magnetic field was tuned between 250 and 550 Gauss with an step size of 30 Gauss and the corresponding in-phase and out-of-phase lock-In amplitudes (X(B),Y(B)) where measured. Figure 5 shows the results for three measurements each averaged over five consecutively taken data sets. The slight off-set in the magnetic field strength between the first measurement and subsequent measurements is due to small permanent magnetization component of the iron core of the electromagnet (hysteresis). Differences in magnetic field strength of $\Delta B$ = *30* Gauss are well resolved both in X(B) and Y(B) which correspond to a difference in the circular birefringence of $\Delta n = 10^{-8}$ and an angular beam deflection of $\Delta\theta = 4.5\ \mathrm{nrad}$. Furthermore the three measurements of each diagram show the same linear response of the system to linear changes of the magnetic field, which is reflected in the slopes of the three data sets.

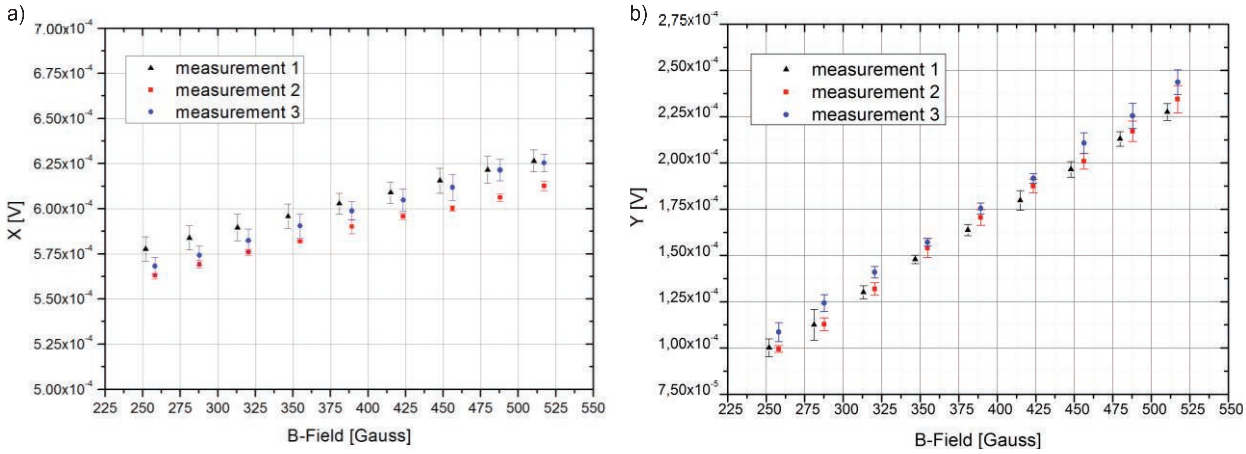


***Figure 5 Measurement of the a ) In-Phase and b) Out-of-Phase Component for variation of the B-Feld***

Moreover have the out-of-phase signals a stronger responding behavior to the changes of the magnetic field strength.

The setup is sensitive to the optical activity (handedness) as is seen in Fig. 6. The B-field was changed in steps of 30-Gauss between $\pm 250$ and $\pm 550$ Gauss. Both the corresponding in-phase component X(B) and the out-of-phase component Y(B) were measured. Only the out-of-phase signal Y(B) changes sign with a reversal of the B-field polarity.
As expected, it is possible to use the out-of-phase component of the signal to determine the absolute sign of the optical activity. All data sets in Fig. 5 and Fig. 6 were recorded with no active temperature or position stabilization of the prism and the PSD, respectively.

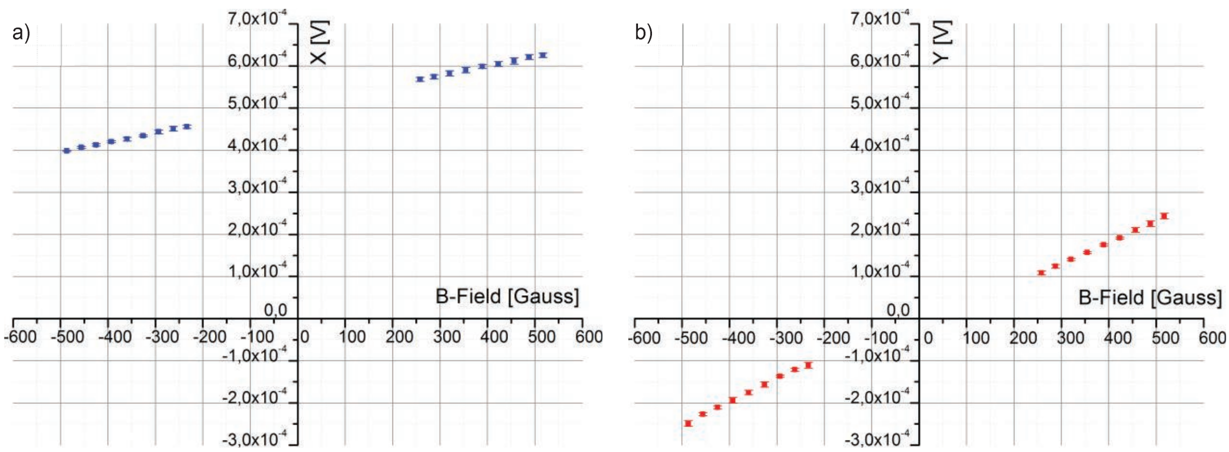


***Figure 6 Measurement of the a) In-Phase and b) Out-of-Phase signal components for changing B-field polarities***

**Literature:**

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