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Determination of Murnaghan constants of plate-shaped polymers under uniaxial tensile load

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Summary

For finite element simulations of mechanic devices the material parameters of each constituent material must be known. Depending on the applied loads to the component, high stresses changing the materials' mechanical behaviour might have to be considered. Therefore measurements are performed under a constant uniaxial tensile stress and the material parameters are identified.

Keywords: ultrasound, acoustoelastic effect, effective elastic constants, Murnaghan constants, polymers

Introduction

Acoustic material parameters are often identified in an inverse measurement procedure, where measurement and simulation results are compared by a cost function while varying the input parameters of the forward model. The forward model can be computed both analytically or numerically assuming Hook's law for small displacements. For greater displacements nonlinearity should usually be considered in the forward model. In this article the material's behaviour under stress is described by the changes in the elasticity matrix, while the forward model itself does not consider the tensile stress. Through these changes the Murnaghan constants [1] are computed.

Measurement setup

During the whole measurement the tensile stress σ_T applied at the specimen (Fig. 1) is kept constant by a programmable logic controller. Then short, high

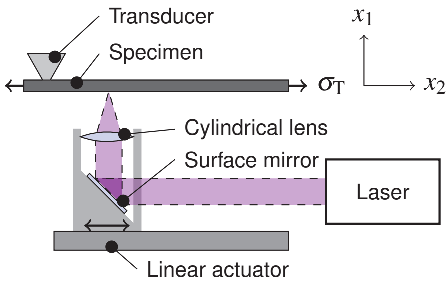

Fig. 1. Measurement system, adapted from [2].

power, laser pulses are focused on the specimens surface to excite acoustic plate waves thermoacoustically, which are received by an ultrasonic transducer. Varying the propagation distance by shifting the optical unit and recording the respective received signals lead to a time and frequency dependent matrix. Application of a two-dimensional Fourier transform results in a matrix depending on frequency and wavenumber, where the propagating modes become visible as ridges.

Material parameter identification

The material identification described in [2] uses a forward model assuming acoustic linearity and plane-strain to compute the plate's eigenfrequencies from given wavenumbers. An optimisation algorithm finds the model's elasticity matrix for which the computed dispersion diagram fits best to the frequency and wavenumber dependent matrix obtained from measurement. Because of the uniaxial load σ_T in x_2-direction (Fig. 1) during the measurements, the material properties change depending on the spatial direction, so that an orthotropic material is assumed in the identification.

Evaluation under constant load

Using the Effective Elastic Constants (EECs) [3] to describe the changes of the elasticity matrix $\mathbf{C}$ the stress-strain relation results in

$$\boldsymbol{\sigma} = (\mathbf{C}_0 + \delta\mathbf{C})\,\boldsymbol{\varepsilon}, \qquad (1)$$

where the entire elasticity matrix consists of the elasticity matrix $\mathbf{C_0}$ measured with a tensile load of $\sigma_\mathrm{T} = 0$ and the change $\delta\mathbf{C}$ under stress $\sigma_\mathrm{T} \neq 0$ w.r.t. $\mathbf{C}_0$. Four elasticity coefficients are identified for each tensile load σ_T:

$$\mathbf{C} = \begin{pmatrix} c_{11} & c_{12} & 0 \\ c_{12} & c_{22} & 0 \\ 0 & 0 & c_{66} \end{pmatrix} \qquad (2)$$

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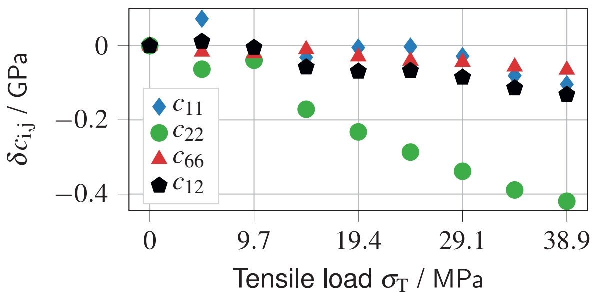

Fig. 2. Absolute change of elasticity coefficients $\delta c_{i,j}$ w.r.t. the elasticity coefficients $c_{0,i,j}$ identified for a tensile load of $\sigma_T = 0$ for specimen 1.

Without any outer stress σ_T the isotropic elasticity matrix is fully described by the Lamé constants $\lambda = c_{12}$ and $\mu = c_{66}$.

Acoustoelastic effect

An initial isotropic material with Lamé parameters λ, μ and density ρ_0 under an uniaxial tensile load is described by

$$\rho_0 v_{11}^2 = \lambda + 2\mu - \frac{\sigma_T}{3\lambda + 2\mu}\left[2l - \frac{2\lambda}{\mu}(m + \lambda + 2\mu)\right] \quad (3)$$

$$\rho_0 v_{22}^2 = \lambda + 2\mu - \frac{\sigma_T}{3\lambda + 2\mu}\left[2l + \lambda + \frac{\lambda + \mu}{\mu}(4m + 4\lambda + 10\mu)\right] \quad (4)$$

$$\rho_0 v_{66}^2 = \mu - \frac{\sigma_T}{3\lambda + 2\mu}\left[m + \frac{\lambda n}{4\mu} + 4\mu\right] \quad (5)$$

with the sound velocities

$$v_{ii} = \sqrt{\frac{c_{ii}}{\rho}}, \quad (6)$$

where v_{11} and v_{22} are the longitudinal sound velocities and v_{66} is the transverse sound velocity [4] of a wave propagating in x_2, polarised in x_1-direction. Knowing the material's Lamé constants and sound velocities the equations are solved to determine the Murnaghan constants m, n and l.

Results

Polycarbonate plates of 3 mm thickness are evaluated exemplarily. Measurements during an applied load up to $\sigma_T = 39\,\text{MPa}$ (8 kN) are performed and the orthotropic elasticity matrix is identified. As Fig. 2 shows, the elasticity parameter c_{22} and so the longitudinal wave velocity in the direction of load σ_T are influenced most significantly, while the elasticity coefficient in x_1-direction (perpendicular to the load) c_{11} changes least. This trend corresponds to the effective elastic moduli determined under constant controlled stress by Brillouin spectroscopy [5]. In general the Murnaghan constants can be determined for each tensile load σ_T. Therefore the mean of the computed Murnaghan constants is shown in Tab. 1. Despite the neglection of nonlinearity, the determined Murnaghan constants are quite similar to the ones from [6], where they were computed from the elasticity tensor coefficients shown in [5], measured through Brillouin spectroscopy. Some differences are expected due to material production tolerances and thus different density and Youngs' modulus for the computation.

Tab. 1. Determined Murnaghan constants m, n and l of polycarbonate.

Specimen	m / GPa	n / GPa	l / GPa
1	-7.8	-21.1	-66.5
2	-7.7	-30.2	-54.3
3	-7.6	-30.9	-52.3
Literature [6]	-12.2	-32	-50

Conclusion

Murnaghan constants are determined by a change of the elasticity coefficients assuming a linear relation. Despite the approximations the values are quite similar to results found in literature. However this influence should still be considered in the forward model in future works. Also acoustic absorption phenomena, temperature and uncertainty influence should be regarded.

Literature

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