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Estimate the error of Offset Issue and Amplitude Mismatch of Atan2 Function

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Summary:
Atan2 function is widely used in most angular sensor nowadays. In order to increase the sensor accuracy, system errors must be compensated before the atan2 calculation. The offset issue and amplitude mismatch are two of the most significant system errors. Until today, different approximation methods are being used to estimate the maximum effects of them. However, these methods are not fully satisfying, especially in terms of high offset issue or angular mismatch, as mentioned. In this paper, a new method with Lissajous-figure is introduced to calculate this impact of those errors analytically and to provide an accurate solution, which can compensate the errors completely.

Keywords: angular sensor, atan2 function, offset issue, amplitude mismatch, Lissajous-figure

Introduction

Rotary position sensors such as resolvers and magnetic sensors are widely used for rotary positioning applications. The most popular method to calculate the angular position is using the atan2 function. In order to improve the sensor accuracy, systematic errors such as offset issue and amplitude mismatch should be compensated. The conventional solutions for them are given in [1]. However, they are not accurate because of the following two facts: a) imperfect sampling (e.g. noise, quantization); b) in some cases corrections are permitted only after atan2 calculation. Furthermore, a more accurate solution for the remaining error is required for sensor designing. Therefore, different methods have been studied: approximation methods for offset issue are introduced in [2] [3] [4] [5] [6]; approximation methods for amplitude mismatch in one-dimensional problem are explained in [5] [6] [7] [8]; a solution for the two-dimensional problem was given in [9]. Those methods base on approximation, so that, they are not accurate enough for precise applications. This paper presents solutions without approximation. Therefore, offset issue and amplitude mismatch can be compensated totally.

Estimate the Impact of Offset Issue

The effect of offset issue can be described using a Lissajous figure (see Fig.1). The circle with solid line represents the Lissajous figure of the ideal sinus and cosine signal, while the circle with dashed line is regarded as the Lissajous figure for signals with offset issues.

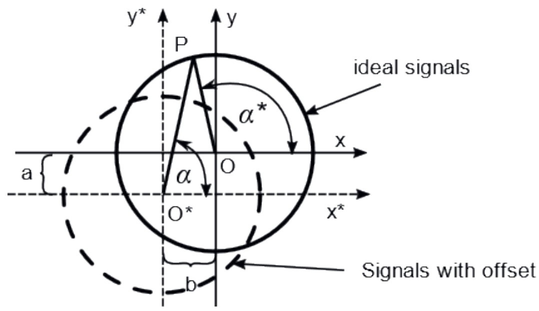

Fig. 1: Lissajous-figure with and without offset issue.

Moreover, the distance a and b represent the offset of sine and cosine signal. Furthermore, the angle α* is equal to the target angle, while the angle α is known as the measured angle. In order to build the relationship between the angles α and α*, both Lissajous-figures should be rotated around their own origins until the axis x and x* are overlapped (see fig. 2a). The rotated angle can be proven as:

$$\varphi^* = \arctan(\tfrac{a}{b}) \quad (1)$$

As both of the x axes overlap, the angle error ε is then the difference between α and angle α*:

$$\varepsilon = \alpha^* - \alpha \quad (2)$$

Furthermore, the angle ε can be calculated with the help of an auxiliary line OH, which is perpendicular to line PO* (see fig 2b) and get:

$$tan(\varepsilon) = \frac{OO^* \cdot sin(\alpha)}{PO^* - OO^* \cdot cos(\alpha)} \quad (3)$$

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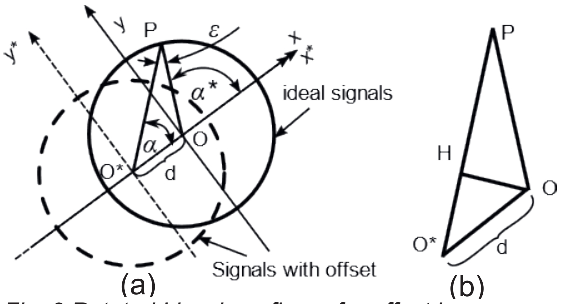

Fig. 2 Rotated Lissajous-figure for offset issue

The both Lissajous-figure must be standardized before the further mathematical deduction. Therefore, the radius O*P is equal to 1, so that the eq. (3) can be simplified as eq. (4), with d being defined as eq. (5).

$$tan(\varepsilon) = \frac{d \cdot sin(\alpha)}{1^* - d^* \cdot cos(\alpha)} \quad (4)$$

$$d = \sqrt{a^2 + b^2} \quad (5)$$

Consequently, the maximum error can be calculated with the help of the eq. (6),

$$\frac{d\varepsilon}{d\alpha} = 0 \quad (6)$$

and get the eq. (7). That means also, if the error reaches its maximum, the edge OP should perpendicular to OO*.

$$\varepsilon_{max} = \arcsin(d) \quad (7)$$

Estimate the Impact of Amplitude Mismatch

The amplitude mismatch can also be illustrated with the Lissajous-figure (see fig. 3a). The Lissajous figure of a system with amplitude mismatch acts as an ellipse instead of a circle (see fig. 3a dashed line).

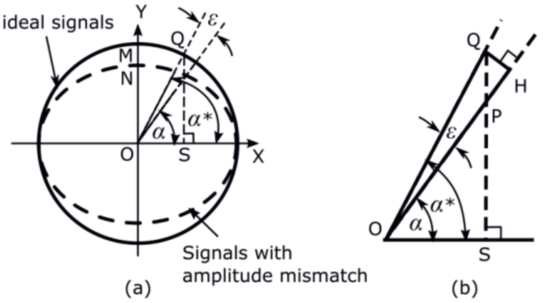

Fig. 3.Lissajous-figure for amplitude mismatch

The amplitude mismatch is formulated as:

$$\gamma = \frac{MO}{NO} \quad (8)$$

The angle α shows the measured angle, while α* corresponds to the angle in an ideal system without amplitude mismatch. Moreover, the angle ε means the deviation between the measured angle and ideal angle. In additional an auxiliary line QH is required, which is perpendicular to OH (see fig. 3b). Observably, the triangle ΔQHP and the triangle ΔOSP are similar. Angle α can be measured and OQ represents the radius with value 1 (after standardization). Furthermore, the quotient of QP and QS is equal to the amplitude mismatch:

$$\frac{QP}{QS} = \gamma \quad (9)$$

Therefore, the edges of triangle OH and QH can be determined with eq. (10) and eq. (11):

$$Oh = Ph + OP = \frac{sin^2(\alpha) + \gamma \cdot cos^2(\alpha)}{\gamma} \cdot OP \quad (10)$$

$$Qh = \frac{1-\gamma}{\gamma} \cdot sin(\alpha) \cdot cos(\alpha) \cdot OP \quad (11)$$

Consequently, the angle error ε is defined as:

$$\tan(\varepsilon) = \frac{(1-\gamma) \cdot sin(2\alpha)}{1 + \gamma + (\gamma - 1) \cdot cos(2\alpha)} \quad (12)$$

The maximum angle error at position α can be calculated as the first derivation. After a simplification the maximum angle is defined as:

$$\varepsilon_{max} = \arcsin\frac{\gamma - 1}{\gamma + 1} = \arcsin(\kappa) \quad (13)$$

Furthermore, that κ means the angle between the eigenvectors of the systems with/without amplitude mismatch (see fig. 4).

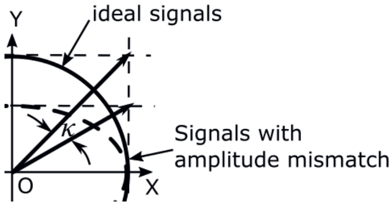

Fig. 4 correlation angle κ

Conclusion

To increase the sensor accuracy mathematical deduction was used. So that the impact of the offset issue and amplitude mismatch can be estimated. The eq. (7) and (13) can be used to estimate the maximum error of the offset issue and amplitude mismatch respectively. With the eq. (4), the offset issue can be compensated completely for each position α. Similarly, the amplitude mismatch can be corrected with the eq. (12) completely.

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