# **Equivalent Circuit Model and Evaluation**of Inductive Conductivity Sensors

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## **Summary:**

Inductive sensors allow for contactless measurement of the electrical conductivity of liquids. A first magnetic coil is used for inducing an eddy current into the liquid, and this current is measured with a second coil. In this contribution, an equivalent circuit model is presented, including also a parasitic direct coupling and the liquid's electrical permittivity. It will be shown analytically and with measurement results that the real part of the ratio of the measurement voltages has to be evaluated and that a trans-impedance amplifier with a small input-impedance has to be used for accurate measurements.

Keywords: Measurement sensor, electrical conductivity, liquid, coil, eddy current

#### Introduction

The measurement of the *electrical conductivity* of liquids is an important task in many industrial processes [1]. In the case of *aggressive* liquids (acids, bases, etc.), preferably *inductive* conductivity sensors are used, because no electrodes are in contact with the liquid [2].

# **Inductive Conductivity Sensors**

Fig. 1 shows the setup of an inductive conductivity sensor, which is *immersed* into the liquid:

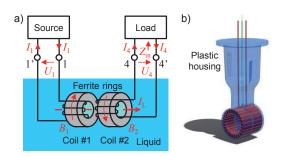


Fig. 1. Inductive conductivity sensor: a) Principle and connection setup, b) sensor design with housing.

The first toroidal coil #1 is driven by a sinusoidal voltage  $U_1$  at an angular frequency  $\omega = 2\pi \cdot f$  (frequency f), resulting in a current  $I_1$  and a magnetic inductance  $B_1$ . The latter induces an eddy current  $I_L$  into the liquid, and this current is flowing through the inner hole of the sensor housing and its surrounding. The eddy current  $I_L$  causes a magnetic inductance  $B_2$  in the second toroidal coil #2, which induces a voltage  $U_4$ . The coil #2 is loaded with a trans-impedance

*amplifier* (with an input-impedance  $Z_{in}$ ), resulting in a current  $I_4 = U_4 / Z_{in}$ .

# **Equivalent Circuit Network**

In Fig. 2, an equivalent circuit network (ECN) of the sensor in Fig. 1 is shown:

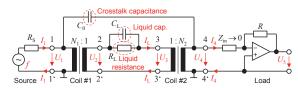


Fig. 2. Inductive conductivity sensor: Equivalent circuit network.

The *electrical conductivity*  $\sigma_L$  of the *liquid*, which is to be measured here, is included in the conductance  $G_{\rm L} = 1 / R_{\rm L} = \sigma_{\rm L} / k_{\rm cell}$  (resistance  $R_{\rm L}$ ) of the liquid, with the so-called 'cell constant'  $k_{\rm cell}$ of the sensor (depending on its geometry). The two coils (numbers of windings  $N_1$  and  $N_2$ , inductances  $L_{11}$  and  $L_{44}$ ) together with the current loop through the conductive liquid are each represented by an ideal, lossless transformer [2]. Furthermore, the ECN in Fig. 2 also includes a parasitic crosstalk capacitance  $C_0$  (direct coupling between the two coils), and the capacitance  $C_{\rm L} = \varepsilon_{\rm L}/k_{\rm cell}$  of the *liquid* (permittivity  $\epsilon_L = \epsilon_{r,L} \cdot \epsilon_0$ , relative permittivity  $\epsilon_{r,L}$ , vacuum permittivity  $\varepsilon_0$ ). Based on Fig. 2, the following ratio  $H_{\rm m}$  between the *output voltage*  $U_5$ =- $R \cdot I_4$  and the *excitation voltage*  $U_1$  is given:

$$H_{\rm m} = \frac{U_5}{U_1} = -R \cdot \frac{\frac{N_2}{N_1} + j\omega \cdot R_{\rm L} \cdot (N_2^2 \cdot C_0 + \frac{N_2}{N_1} \cdot C_{\rm L})}{Z_{\rm in} \cdot (1 + j\omega \cdot R_{\rm L} \cdot C_{\rm L}) + R_{\rm L} \cdot N_2^2 \cdot (1 + j\omega \cdot Z_{\rm in} \cdot C_0 + Z_{\rm in} / (j\omega \cdot L_{44}))}$$
 (1)

The concept of the sensor is to measure the voltages  $U_5$  and  $U_1$  and to assess their ratio  $H_{\rm m}$ in order to estimate the resistance  $R_{\rm L}$  of the *liquid* (and so the conductivity  $\sigma_{\rm I}$ ) by means of (1). However, the parameters  $L_{44}$ ,  $C_0$ , and also Zin can significantly change over different samples of the same type of sensor, and also over temperature, aging, etc. For this reason, these parameters and also the unknown  $C_{\rm L}$  have to be eliminated from the measurement (compare [1]). This can be achieved by using a transimpedance amplifier with a sufficiently small input-impedance  $Z_{in}$ , a sufficiently *large* angular frequency  $\omega$ , and a *large* inductance  $L_{44}$  giving 
$$\begin{split} |Z_{\rm in}| &<< \omega \cdot L_{44}, \quad \text{and} \quad |Z_{\rm in}| << 1 \, / \, (\omega \cdot C_0), \quad \text{and} \\ |Z_{\rm in}| &<< R_{\rm L} \cdot N_2^{\ 2} / |1 + j \omega \cdot R_{\rm L} \cdot C_{\rm L} \ |. \ \text{Under these condition} \end{split}$$
tions,  $H_{\rm m}$  is given as follows, see (1):

$$H_{\rm m} = \frac{U_5}{U_1} = -R \cdot \left( \frac{1}{N_1 \cdot N_2 \cdot R_{\rm L}} + j\omega \cdot (C_0 + \frac{C_{\rm L}}{N_1 \cdot N_2}) \right) \tag{2}$$

The result in (2) shows that  $R_{\rm L}$  is now directly accessible from the *real part*  ${\rm Re}(H_{\rm m})$  of the voltage ratio  $H_{\rm m}$ , scaled by the *numbers of windings*  $N_1$  and  $N_2$  of the two coils:

$$Re(H_{\rm m}) = Re\left(\frac{U_5}{U_1}\right) = -\frac{1}{N_1 \cdot N_2} \cdot \frac{R}{R_1}$$
(3)

Because of this scaling in (3), inductive sensors are preferably used for liquids with large conductivities  $\sigma_L$  (i.e. with small resistances  $R_L$ ). In contrast, conductivity sensors with Galvanic coupling are to be preferred for liquids with small conductivities [1].

## **Experimental Evaluation**

Measurements have been performed with a sensor (with  $k_{\text{cell}} = 6.25 \text{ cm}^{-1}$ ,  $N_1 = N_2 = 25$ ,  $L_{11} = L_{44} = 12 \text{ mH}, f = 2.5 \text{ kHz}, R = 1 \text{ k}\Omega$ ) in order to verify the findings above and to evaluate the achievable measurement accuracy and range. The voltage ratio  $H_m$  has been directly measured with a calibrated vector network analyzer (model Bode 100; OMICRON electronics GmbH, Klaus, Austria). For measurements under known and well-defined conditions, the liquid has been replaced by a metallic wire (as current loop) through the inner hole of the sensor. The loop has been loaded with a changing *lumped* resistance  $R_L = 12 \Omega ... 220 k\Omega$ . For each different  $R_{\rm L}$ , an according nominal *given* conductivity  $\sigma_L = k_{cell}/R_L$  has been calculated. In Fig. 3, the measured nominal conductivity  $\sigma_L$  over the varying given conductivity is shown for both, using the mag*nitude*  $|H_{\rm m}|$  of the voltage ratio  $H_{\rm m}$  only and for using the *real part*  $Re(H_m)$  of  $H_m$ :

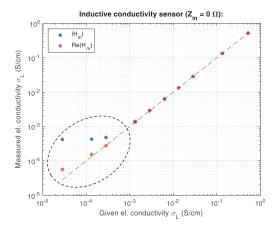


Fig. 3. Measured nominal conductivity: Magnitude and real part of voltage ratio (for  $Z_{\rm in}$  = 0  $\Omega$ ).

The results confirm that the measurement range of the sensor can be largely extended towards small conductivities by using the *real part* of the voltage ratio  $H_{\rm m}$  and a *small* input-impedance  $(Z_{\rm in} \rightarrow 0)$  in order to eliminate the influence of the unknown parasitic elements. In Fig. 4, same plots as in Fig. 3 are shown, but now for a *large* input-impedance  $Z_{\rm in} = 330 \, \Omega$ ):

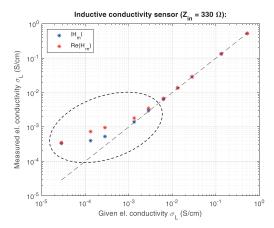


Fig. 4. Same as in Fig. 3 (but for  $Z_{in} = 330 \Omega$ ).

It can be seen that the measurement error is now, as expected, much larger with both, using the real part and the magnitude of  $H_{\rm m}$ . In conclusion, a *trans-impedance amplifier* with a *small input-impedance* has to be used, and the *real part* of the voltage ratio has to be evaluated to guarantee for precise measurements over a large measurement range.

### References

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