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Propagation of uncertainty for an Adaptive Linear Approximation algorithm

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Summary:

In machine learning, a variety of algorithms are available for feature extraction. To obtain reliable features from measured data, the propagation of measurement uncertainty is proposed here in line with the Guide to the Expression of Uncertainty in Measurement (GUM). Recently, methods for the discrete Fourier and Wavelet transform have been published. Here, the Adaptive Linear Approximation (ALA) as a further complementary feature extraction algorithm is considered in combination with an analytical model for the uncertainty evaluation of the ALA features.

Keywords: measurement uncertainty, uncertainty propagation, feature extraction, Adaptive Linear Approximation, machine learning

Motivation

One of the most important advances in sensor technology has been the development of smart sensors. These sensors carry out internal signal processing, e.g. for machine learning, in addition to data acquisition. For data analysis with smart sensors, a fully automated machine learning toolbox (see Fig. 1) has been developed [1] which can be used without any expert knowledge and without knowledge of a physical model of the process. In this toolbox, five complementary algorithms for feature extraction (FE) and three for feature selection (FS) are combined and both classification based on the best combination of FE/FS and validation can be carried out. For the FE, Adaptive Linear Approximation, Principal Component Analysis and the first four statistical moments are used to extract features in the time domain. For extracting features in the frequency domain, the Best Fourier Coefficient method is used and for the time-frequency domain the Best Daubechies Wavelet method is applied. In this unsupervised step, as many features as possible are extracted. After that, a supervised feature selection is performed either with simple Pearson correlation to the target or complex methods, i.e. Recursive Feature Elimination Support Vector Machine or RELIEFF. The objective of this step is to concentrate as much information as possible in as few features as possible and to remove

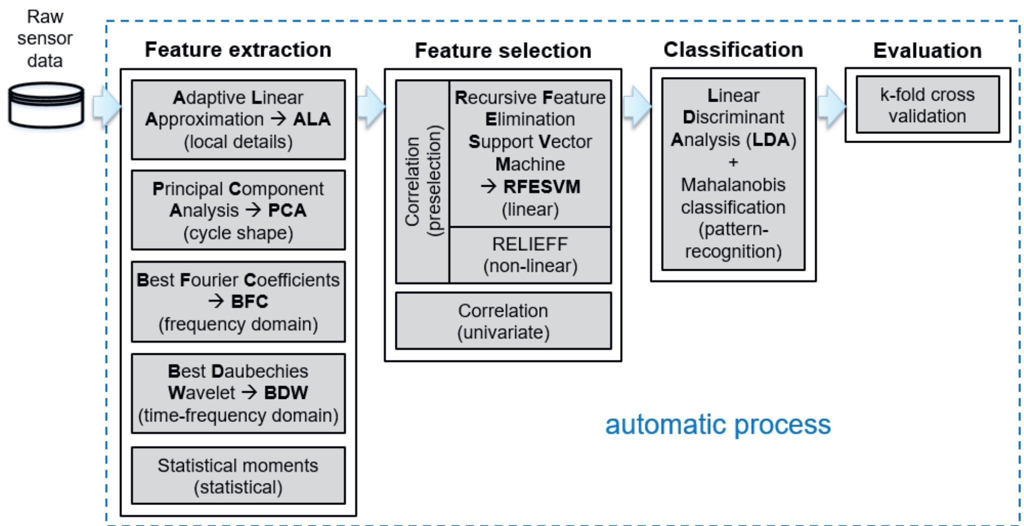

Fig. 1 Schematic of the algorithms implemented in the software toolbox [2]

features with low information content and redundant features from the set of features extracted in the previous step.

However, none of the methods within the toolbox so far consider measurement uncertainty. Recently, propagation of uncertainties for Discrete Fourier transform (DFT) [4] and Discrete Wavelet decomposition (DWT) [to be published soon] have been proposed. In this contribution, the propagation of uncertainties in line with the *Guide to the Expression of Uncertainty in Measurement (GUM)* [3] is applied to feature extraction with *Adaptive Linear Approximation (ALA)*.

Results

ALA approximates a certain time segment of sensor data or a measurement cycle [1] with linear segments of variable length. The mean and the slope of each segment are extracted as features. Dividing the cycle into many segments leads to many features together with a small approximation error and vice versa. When there is no significant decrease of the approximation error when performing an additional split, the algorithm stops automatically. In Fig. 2, the approximation of an original measurement cycle by ten segments is shown.

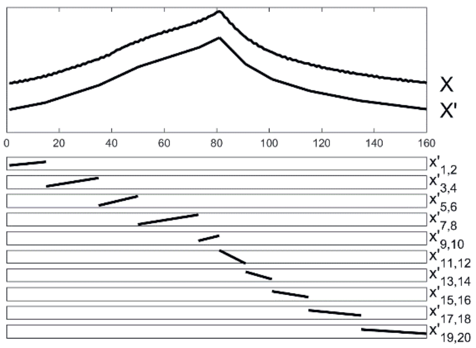

Fig. 2. Approximation X' by mean values (uneven indices) and slopes (even indices) of ten segments determined by ALA compared to the original cycle X (shifted for better clarity).

Since the calculations below are the same for every cycle, they are shown here for one cycle only. Let $Y = (y_1, \dots, y_n) \in \mathbb{R}^{1\times n}$ denote the real-valued time-domain signal, i.e. one cycle. The result of the ALA for this cycle Y is given by

$$F = \left(\overline{y_1}, \dots, \overline{y_{u_3+1}}, b_1, \dots, b_{u_3+1}\right) \in \mathbb{R}^{1\times 2(u_3+1)},$$

where $\overline{y_\mathrm{k}}$ denotes the mean value and b_k the slope of the k-th segment, respectively, of a cycle for $k = 1, \dots, u_3 + 1$. u_3 is the number of splits and therefore, $u_3 + 1$ the number of segments into which the cycle is split.

The mean value and slope for the k-th segment are determined by

$$\overline{y_k} = f(y_i) = \frac{1}{v_{k+1} - v_k + 1} \sum_{i=v_k}^{v_{k+1}} y_i$$

and

$$b_k = h(y_i) = \frac{\sum_{i=v_k}^{v_{k+1}} (t_i - \overline{t_k})(y_i - \overline{y_k})}{\sum_{i=v_k}^{v_{k+1}} (t_i - \overline{t_k})^2}.$$

For the propagation of uncertainties according to GUM, the sensitivities of the mapping $Y \mapsto F$ are given by

$$c_{k,j} = \frac{\partial \overline{y_k}}{\partial y_j} = \frac{1}{v_{k+1} - v_k + 1}$$

and

$$d_{k,j} = \frac{\partial b_k}{\partial y_j} = \frac{t_i - \overline{t_k}}{\sum_{i=v_k}^{v_{k+1}} (t_i - \overline{t_k})^2},$$

for $j = v_k, \dots, v_{k+1}$. Thus, the sensitivity matrix has a block structure and is given by

$$\mathbf{J}^{\mathbf{m}}_{\bar{\mathbf{y}},\mathbf{b}} = \begin{pmatrix} \mathbf{C} \\ \mathbf{D} \end{pmatrix} \in \mathbb{R}^{2(u_3+1)\times n}.$$

In the sensitivity matrix, the matrix $\mathbf{C} \in \mathbb{R}^{(u_3+1)\times n}$ denotes the upper submatrix and has the entries $\left(c_{k,j}\right)_{k=1,\dots,u_3, j=v_k,\dots,v_{k+1}}$. For the submatrix $\mathbf{D} \in \mathbb{R}^{(u_3+1)\times n}$, simply replace $\mathbf{C}$ by $\mathbf{D}$ in the statement above.

The given covariance matrix of the input quantities $\mathbf{U_y} \in \mathbb{R}^{\mathrm{n}\times\mathrm{n}}$ leads to the following expression for the covariance matrix $\mathbf{U_F} \in \mathbb{R}^{\mathrm{n}\times\mathrm{n}}$ associated with F:

$$\begin{aligned} \mathbf{U_F} &= \mathbf{J}^{\mathbf{m}}_{\bar{\mathbf{y}},\mathbf{b}} \cdot \mathbf{U} \cdot \mathbf{J}^{\mathbf{m}}_{\bar{\mathbf{y}},\mathbf{b}} \\ &= \begin{pmatrix} \mathbf{C}\mathbf{U_y}\mathbf{C^T} & \mathbf{C}\mathbf{U_y}\mathbf{D^T} \\ \left(\mathbf{C}\mathbf{U_y}\mathbf{D^T}\right)^T & \mathbf{D}\mathbf{U_y}\mathbf{D^T} \end{pmatrix}. \end{aligned}$$

This block structure of the covariance matrix $\mathbf{U_F}$ can be used to deal with computer memory issues. Since $\mathbf{U_F}$ is symmetric, only three blocks must be stored, see also [4].

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