DOI 10.5162/EUROSENSORSXXXVI/OT1.5

The Optimal Planform of a Cantilever Unimorph Piezoelectric Vibrating Energy Harvester (PVEH), with a Device-Layer Edge Block

Eliya Salman[1]*, Sahar Lustig*[1]*, David Elata*[1]
[1] *Technion – Israel Institute of Technology, Faculty of Mechanical Engineering, Haifa, 32000, Israel*

elata@technion.ac.il

Summary:
We consider the planform of a cantilever piezoelectric vibrating energy harvester (PVEH), that has an edge block which is patterned *only* in the device layer. We derive an analytic expression of the planform, which ensures an optimal performance of the cantilever PVEH. The contours of the optimal planform are described by Bessel functions, and the predictive capabilities of our new model are demonstrated by the good agreement with finite elements simulations.

Keywords: Piezoelectric unimorph, energy harvesting, vibrating cantilever, uniform strain, uniform curvature.

Background, Motivation and Objective

The optimal performance of a cantilever piezoelectric vibrating energy harvester (PVEH), is achieved when the axial strain in the piezoelectric layer, is uniform. Technology dictates the construction and thickness of the unimorph, but its planform is a design choice which makes optimization possible.

In a recent paper [1] we considered a PVEH with a massive edge block that extended into the handle layer. The inertia of this edge block dominated the vibration response of the structure. We showed that for such a massive edge block the optimal planform of the PVEH is a *trapeze*. The same conclusion was presented in many previous studies which were based on experiments, simulations, or a combination of both (e.g. [2],[3]). However, our work was the first rigorous analysis of the of the problem, and we presented an explicit functional form of the optimal trapeze planform [1]. That model was the first ever to offer predictive capabilities.

In contrast to those devices, there is a different class of PVEHs in which the edge block is patterned *only* in the device layer (e.g. Fig. 1), and does not extend into the underlaying handle layer. In this case, the inertia of the beam is as important as the inertia of the edge block.

In the present study we show that the optimal planform of a cantilever PVEH with a *device-layer* edge block, is *not* a trapeze with straight contours, but rather it is a planform with curved contours described by *Bessel* functions. We validate or model and demonstrate its predictive capabilities by using finite elements simulations.

Modelling – Analytic derivation

Figure 1 presents a schematic illustration of the PVEH. The beam has a length L, a uniform thickness h, and a rectangular cross-section with width $w(x)$, that changes from the clamped edge at $x=0$ to the far edge at $x=L$. It is assumed that the Euler-Bernoulli beam theory is applicable. An edge block which is patterned in the device layer is connected to the far edge (i.e. $x=L$), and has a mass m_L and a moment of inertia I_L. The moment of inertia refers to the z axis, and it is given relative to the center point of the edge cross-section (i.e. $x=L$, $y=z=0$).

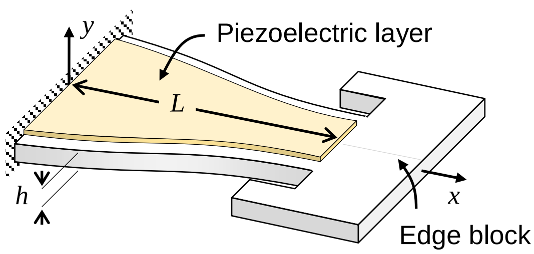

Fig. 1. PVEH with a device-layer edge block.

Within the context of the Euler-Bernoulli beam theory, we consider a steady state vibration of the cantilever in the form $y(x,t)=Y(x)\cdot T(t)$, where $Y(x)$ is the mode shape of the transverse deflection. The aim of the present study is to determine the functional form of $w(x)$ such that the axial strain on the top surface is uniform

$$\varepsilon_{xx}(x,y,z=h/2)=-\frac{d^2Y(x)}{dx^2}\frac{h}{2}=\bar{\varepsilon} \qquad (1)$$

The governing equation that determines the out-of-plane displacement $y(x,t)$, is given by

$$\rho h w(x)\frac{\partial^2 y(t,x)}{\partial t^2}=-E\tilde{I}_{yy}\frac{\partial^2}{\partial x^2}\left(w(x)\frac{\partial^2 y(t,x)}{\partial x^2}\right) \qquad (2)$$

where ρ is the density of the elastic material and E is the Young modulus, and $\tilde{I}_{yy}=h^3/12$.

Extracting $Y(x)$ from eq. (1) and substituting it into eq. (2), yields the differential equation

$$\frac{d^2w(x)}{dx^2}-\frac{\rho h}{2E\tilde{I}_{yy}}\omega^2x^2w(x)=0 \qquad (3)$$

Equation (3) gives the optimal planform for a uniform strain over the top surface of the PVEH

$$w(x)=a_1\sqrt{x}\cdot I_{0.25}(\beta)+a_2\sqrt{x}\cdot K_{0.25}(\beta) \qquad (4)$$

Here $\beta=x^2\omega\sqrt{\rho h/8E\tilde{I}_{yy}}$, $I_{0.25}$ and $K_{0.25}$ are modified *Bessel* functions, and the constants a_1 and a_2 are determined from the boundary conditions related to the resultant shear force and bending moment at $x=L$.

Finite Elements Simulations

With energy harvesting applications in mind, we consider cantilever beams made from single-crystalline silicon (SCS) using silicon on insulator (SOI) wafers technology [3]. Specifically, the beam and the edge block are patterned in the device-layer of the wafer. We consider beams that are along the (110) orientation of the anisotropic SCS where E_{xx}=169.7 GPa, and typical values of h=10 μm, L=2000 μm and a vibration frequency of ω=30,000 Rad/s (4.774 kHz). All edge-blocks are rectangular, with width w_{EB}=200 μm, and lengths L_{EB}=100, 200, 300 and 400 μm.

The eigenfrequency of the cantilever beam with a device-layer edge block was simulated using the COMSOL™ 6.0 finite elements code [4]. The *nonuniformity*, $S_{\varepsilon_{xx}}$, of the axial strain ε_{xx}, over the top surface of the cantilever, is computed by

$$S_{\varepsilon_{xx}}=\sqrt{\int_{x=0}^{L}\varepsilon_{xx}^{2}w(x)dx\Big/\int_{x=0}^{L}|\varepsilon_{xx}|w(x)dx-1} \qquad (5)$$

For a uniform strain, the nonuniformity is identically zero, $S_{\varepsilon_{xx}}$=0.

Table 1 presents the results for the different edge block lengths, L_{EB}. The strain nonuniformities are rather small. For perspective, we consider a non-optimal trapezoidal cantilever beam, with the same widths at the clamped and the far edge, as in the third case in table 1 (L_{EB}=300 μm). Figure 2 compares the strain distribution over the top surface for the optimal planform, and for the non-optimal trapezoidal planform. The strain over the top surface of the optimal beam (Fig. 2 bottom) is more uniform by a factor of ~50 relative to that of the trapezoidal beam (Fig. 2 top).

Tab. 1: The results of the eigenfrequency simulations for different edge blocks lengths

L_{EB} μm	Rel error of ω_n [%]	Nonuniformity $S_{\varepsilon_{xx}}$
100	0.023	$2.5851\cdot10^{-5}$
200	0.151	$8.7729\cdot10^{-5}$
300	0.435	$3.0367\cdot10^{-4}$
400	1.001	$9.8769\cdot10^{-4}$

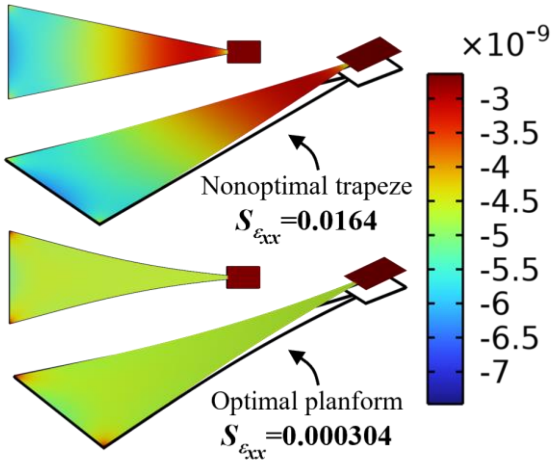

Fig. 2. Comparison of the strain distribution over the top surface between the optimal planform (bottom) and the trapezoidal planform (top).

This validates the predictive capabilities of the new model in Eq. (4).

Acknowledgement: This work was supported by the Israel Science Foundation, Grant 1820/22.

References

[1] E. Salman, et al., "On the optimal planform of a cantilever unimorph piezoelectric vibrating energy harvester," Smart Mater. Struct., **33**(3), 35029, 2024, doi: 10.1088/1361-665X/ad28d0.

[2] E. Halvorsen and T. Dong, "Analysis of tapered beam piezoelectric energy harvesters," PowerMEMS 2008, Sendai, 2008, pp. 241–244.

[3] Y. Jia and A. A. Seshia, "Five topologies of cantilever-based MEMS piezoelectric vibration energy harvesters: a numerical and experimental comparison," Microsyst. Technol., **22**(12), 2841–2852, Dec. 2016, doi:10.1007/s00542-015-2599-z.

[4] COMSOL, www.comsol.com., COMSOL AB, Stockholm, Sweden.