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Avoided mode-crossing in MEMS resonators under different dissipation mechanisms

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Summary: This study investigates the phenomenon of avoided-crossing in MEMS resonators and its impact on the quality factor (Q-factor) under two key dissipation mechanisms: anchor losses and fluid-structure interaction. Numerical and experimental results reveal significant reductions in Q-factors during mode crossings and avoided-crossings, particularly in scenarios involving width variations and fluid density changes. These findings highlight the need to consider avoided mode-crossing in MEMS resonators for obtaining high Q-factors and hence, high device sensitivity as well as predictable bandwidth.

Keywords: MEMS, Avoided-crossing, Anchor losses, Fluid-structure interaction

Background, Motivation and Objective

MEMS are ubiquitous components in modern technology, integral to applications in sensing, communication, and energy harvesting [1]. MEMS resonators are characterized by the resonance frequency and quality factor (Q-factor) of their vibrational modes. When a parameter of the resonator or its surrounding environment changes, the resonance frequencies and corresponding vibrational modes also shift. A particularly intriguing phenomenon occurs when two resonance frequencies converge closely: vibrational modes often degenerate, and resonance frequencies "veer" apart in a phenomenon known as avoided-crossing or mode-veering [2, 3].

Understanding multimode effects, such as avoided-crossing and mode localization [4], is critical for optimizing MEMS resonators. Research has predominantly focused on changes in resonance frequencies and vibrational modes, leaving its impact on the Q-factor underexplored. This study addresses this gap by presenting a theoretical framework as well as experimental results to investigate the influence of avoided-crossing on the Q-factor in two distinct scenarios: anchor losses and fluid-structure interaction.

Description of the New Method or System

The Q-factor Q is defined as the ratio of stored energy in a structure to dissipated energy over an oscillation period. As different damping mechanisms contribute to the dissipated energy, we focus on anchor losses and fluidic losses. Anchor losses in MEMS resonators arise from the energy radiating into the substrate through the mechanical supports, leading to energy dissipation and a reduction in the Q-factor. To model and quantify these losses, we employ the Finite Element Method (FEM) with the governing equation as the linear elasticity in the absence of external forces

$$\nabla \cdot \sigma + \rho\omega^2 \mathbf{u} = \mathbf{0}, \qquad (1)$$

where σ represents the Cauchy stress tensor, ρ is the density, ω is the angular frequency and $\mathbf{u}$ is the displacement tensor. The system is solved incorporating a Perfectly Matched Layer (PML) to simulate the energy absorption in the substrate effectively, yielding a complex eigenvalue $\tilde{\omega}$ problem from which the Q-factor is determined as

$$Q_{\mathrm{anchor}} = \frac{\mathrm{Re}(\tilde{\omega})}{2\mathrm{Im}(\tilde{\omega})}. \qquad (2)$$

For the fluidic Q-factor, we adopt a semi-numerical approach based on the Kirchhoff plate equation coupled with the Stokes flow for fluid dynamics as

$$\frac{h^3}{12} C_{\alpha\beta\gamma\delta}\phi_{,\alpha\beta\gamma\delta} - \omega^2\rho h\phi = P_{\mathrm{ext}} + P_z, \qquad (3)$$

where $C_{\alpha\beta\gamma\delta}$ are the components of the fourth-order elasticity tensor. P_{ext} is a driving force and P_{hydro} is the hydrodynamic point force [5]. This equation is solved with Galerkin mode decomposition (GMD), which efficiently determines the spectral displacement ϕ of wide resonators in viscous fluids, from which Q and the resonance frequencies are obtained.

Results

We investigate avoided mode-crossing by considering a cantilevered silicon microplate with length equals 200 $\mu\mathrm{m}$, thickness equals 15 $\mu\mathrm{m}$ and width varying from 250 $\mu\mathrm{m}$ to 500 $\mu\mathrm{m}$. Selected resonance frequencies and Q-factors are shown in Fig. 1. Those are for the out-of-plane modes 1:3 and 2:1, as well as the lateral mode. At a width of 355.5 $\mu\mathrm{m}$, the modes

2:1 and 1:3 are subject to an avoided-crossing phenomenon, where the resonance frequencies veer apart instead of simply crossing over each other. Crucially, note that avoided-crossing is accompanied by a significant decrease in the Q-factor of the 1:3 mode, as seen on Fig 1b. A different mode crossing occurs between the lateral and 1:3 modes at a width of 316.5 μm where no veering of the resonance frequencies is observed. Neverthelles, the mode crossing results in a pronounced reduction in the Q-factor for the 1:3 mode from $Q \approx 10^6$ to $Q \approx 10^3$. To confirm these observations, experiments were performed, whose results for the 1:3 mode are shown in Fig. 1c, where similar drops in Q are seen in $w \approx 365$ μm and $w \approx 320$ μm due to the crossing of the resonance frequencies of the 1:3 mode with other modes.

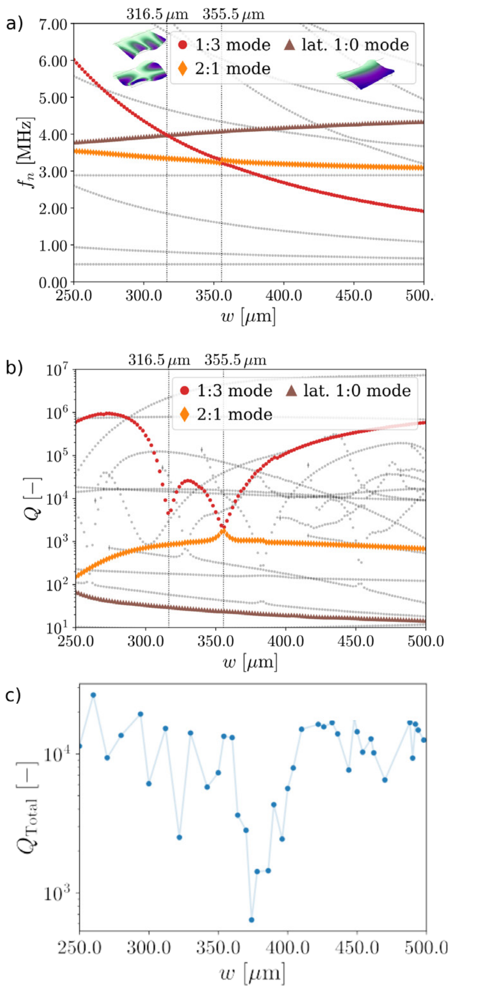

Fig. 1: a) Resonance frequencies and b) Q-factors (right) of MEMS resonator modes as a function of resonator width, varying from 250 μm to 500 μm. c) Experimentally measured Q-factor of the 1:3 mode showing similar drops at $w \approx 365$ μm and $w \approx 320$ μm.

In addition to the anchor losses analysis, we also consider fluidic damping. The resonance frequencies and Q-factors of the 5:0 and 3:2 modes were analyzed for a silicon microplate as a function of fluid density ranging from 1 kg/m³ to 1000 kg/m^3. Note that a notable increase in the Q-factor in the 3:2 mode occurs at a fluid density of approximately 10 kg/m^3 as shown in Fig. 2. This sudden change indicates an interaction with another mode, where the energy dissipation mechanism of mode 3:2 is altered through the fluid-structure coupling.

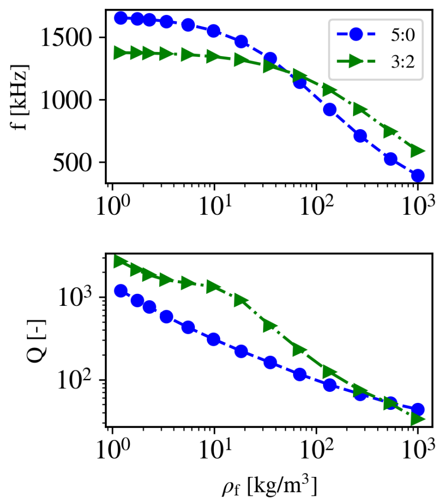

Fig. 2: Resonance frequencies (top) and Q-factors (bottom) of the 5:0 mode (blue circles) and 3:2 mode (green triangles) as a function of fluid density.

These findings demonstrate the sensitivity of MEMS resonator's Q-factors to avoided-crossing.

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