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On Uncertainty Evaluation using Virtual Experiments

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Summary:
Virtual experiments have become increasingly important in metrology and industry. Combined with Monte Carlo methods, they are also employed for uncertainty evaluation. However, the modeling principles underlying the *Guide to the Expression of Uncertainty in Measurement* (GUM) generally differ from the concepts of a typical virtual experiment. We discuss these conceptual differences and exemplify how they can affect resulting uncertainties. We also show that for certain linear models virtual experiments can nevertheless be used to determine measurement uncertainties in line with the GUM.

Keywords: measurement uncertainty, virtual experiments, GUM, Monte-Carlo Method, linear models

Virtual experiments and the GUM

A virtual experiment is typically a numerical model of a measurement process which produces virtual data whose properties reflect those of the data observed in the real experiment. Virtual experiments have become increasingly important in modern metrology and industrial applications, e.g., to explore the accuracy of a measurement device, to specify machine tolerances needed to reach a required accuracy, or to identify significant sources of uncertainty. Combined with Monte-Carlo methods, virtual experiments have been proposed for the evaluation of measurement uncertainties [1].

However, the metrological standard for uncertainty evaluation specified in the GUM [2] does not rely on a simulation of the measurement process but rather uses a model for which one of its input quantities is represented by the outcome of the measurement process. This different role of input and output quantities for a GUM model and a typical virtual experiment is illustrated in Fig. 1.

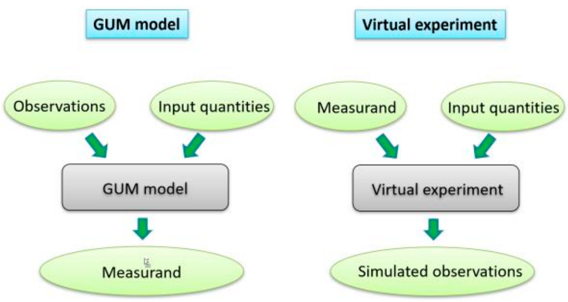

Fig. 1. Comparison of GUM model and virtual experiment.

Comparison in terms of a linear model

The difference of the two approaches is shown for a model as used in the GUM, which represents an almost direct measurement of a measurand y

$$y = x + z^3. \qquad (1)$$

Here x is a quantity for which repeated observations are given (GUM Type A information) and z a quantity with Type B information. The variance σ^2 of the distribution from which repeated observations are taken is assumed to be known. The corresponding model for the virtual experiment is

$$x_{\mathrm{VE}} = y_0 - z^3 + \varepsilon\,, \qquad (2)$$

where ε models the random error with variance σ^2 observed in repeated measurements, and y_0 is a fixed value selected for the simulated measurand. The subscript VE denotes the outcome of the virtual experiment. Each time the virtual experiment is run, different values for ε and z are taken, where a value for z is drawn from the probability density function (PDF) that encodes the knowledge about z (Type A information).

The blue line in Fig. 2 shows the PDF for the measurand y which has been obtained by applying the GUM-S1 Monte-Carlo method [3] to measurement model (1). The corresponding PDF of randomly drawn virtual data x_{VE} (red line) via the virtual experiment (2) was determined by application of a Monte-Carlo method. As can be seen, the two distributions obtained by GUM-S1 and by the virtual experiment clearly differ with respect to the location of their mean values and, moreover, also with respect to their shape. That is, simply shifting the PDF obtained by the virtual experiment does not yield the PDF for the measurand produced by the application of GUM-S1.

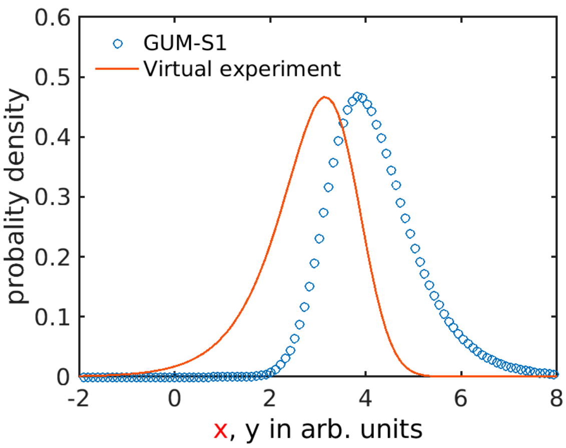

Fig. 2. Comparison of the PDFs obtained by GUM-S1 and by generation of random drawn virtual data using the virtual experiment.

GUM uncertainty evaluation using virtual experiments

While the GUM (and GUM S1) uncertainty evaluation follows rather strict rules this is not the case when applying Monte-Carlo to a virtual experiment. In fact, a Monte-Carlo virtual experiment can be carried out following rather different strategies which may even lead to markedly different resulting uncertainties as demonstrated in [4].

However, for linear models, a simple transformation of the virtual experiment (2) can be applied, such that the PDF produced by repeatedly running the procedure equals an application of GUM-S1, cf. [5]. More specifically, this can be achieved by a transformation according to

$$y = x + y_0 - x_{\mathrm{VE}}\,, \qquad (3)$$

where x denotes the real observation, y_0 the chosen virtual measurand, and x_{VE} the random outcome of the virtual experiment (2). By repeatedly running the virtual experiment and applying the transformation (3), the resulting samples equal those obtained by applying the Monte Carlo approach of GUM-S1. Fig. 3 illustrates this equivalence for the considered models and chosen PDF for z.

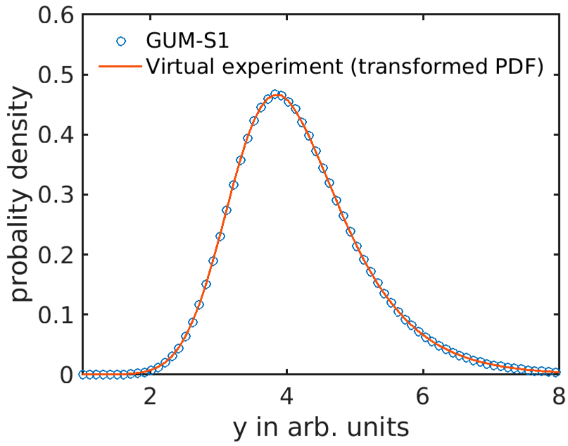

Fig. 3. Comparison of the PDFs obtained by GUM-S1 and by transformation (3) applied to the randomly drawn virtual data.

Conclusions

Virtual experiments can be helpful to develop and assess uncertainty evaluations within the framework of the GUM. For specific linear models, a simple transformation of randomly drawn virtual data yields a GUM-compliant uncertainty analysis. For nonlinear models, however, the distribution of randomly drawn virtual data can no longer be easily transformed into a GUM-compliant uncertainty evaluation. The development of corresponding approaches will be the topic of future research.

References

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