

# An Optical Stokes Absolute Roll-angle Sensor with a Full Measurement Range of 360°

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## Summary:

Angle measurements (roll, pitch, and yaw) are important parameters in many different fields. Roll angle is deemed the most challenging measurement quantity because the angular displacement of the roll angle is perpendicular to the probe beam. In this work, we proposed a Stokes roll-angle sensor whose measurement range can achieve 360° without ambiguity. The principle is based on the polarization change of a sensing unit (vortex retarder). Within the whole measuring range, the measurement errors are less than  $\pm 0.15^\circ$  and the standard deviations are less than  $0.09^\circ$ .

**Keywords:** Roll angle, angle sensor, polarization sensor, Stokes polarimeter, angle measurement

## Background and Motivation

Angle measurements (roll, pitch, and yaw angles) are important parameters in many different fields, e.g., remote sensing [1], navigation [2] and object tracking [3]. Roll angle is deemed the most challenging measurement quantity because the angular displacement of the roll angle is perpendicular to the probe beam. Most general methods for roll-angle measurements are variation of polarization states, rotary encoder, autocollimator, optoelectronic level, relative position shift of two parallel laser beams [4]. Nevertheless, only the first two methods can measure large angular displacement. The rest methods are limited to few degrees or few arcminutes. Compared to variation of polarization states, rotary encoders require a fixed distance between the optical head and the disk scale. This drawback decreases the feasibility of long-range measurements. The polarization-based measurements for the roll angle can be used for remote sensing but the range is limited to 180° [1,5,6].

In this work, we propose a Stokes roll-angle sensor whose measurement range can achieve 360° without ambiguity. The measurement principle is based on the special polarization characteristic of a vortex quarter-wave retarder (VR). The roll angle is acquired by measuring the change of the polarization states of linearly polarized light after passing the VR.

## Description of the New Method

Figure 1 shows the schematic of the proposed roll-angle sensor. The light source passes a linear polarizer (LP) and a VR and the beam was received by a Stokes polarimeter.

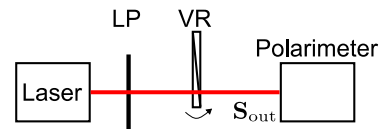


Fig. 1. The schematic of the proposed roll-angle sensor.

In the system, the VR is a sensing unit for the roll angle and the other components are fixed. There is an offset between the center of the VR and the light beam. The fast axis of the VR (first order) rotates continuously and its Mueller matrix is shown as

$$\mathbf{M}_{VR} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2 \phi & \frac{1}{2} \sin 2\phi & \sin \phi \\ 0 & \frac{1}{2} \sin 2\phi & \sin^2 \phi & -\cos \phi \\ 0 & -\sin \phi & \cos \phi & 0 \end{bmatrix},$$

where  $\phi$  is the orientation of the fast axis. If the axis of the LP is horizontal, the measured Stokes vector  $\mathbf{S}_{out}$  can be expressed as

$$\mathbf{S}_{\text{out}} = \begin{bmatrix} 1 & \cos^2 \phi & \frac{1}{2} \sin 2\phi & -\sin \phi \end{bmatrix}^T.$$

Figure 2 presents simulated Stokes parameters for the roll angle from  $0^\circ$  to  $360^\circ$ . It is obvious that the periods of  $s_1$ ,  $s_2$ , and  $s_3$  are  $180^\circ$ ,  $180^\circ$  and  $360^\circ$ , separately. Therefore, the proposed sensor can measure roll angles with an unambiguous measurement range of  $360^\circ$ .

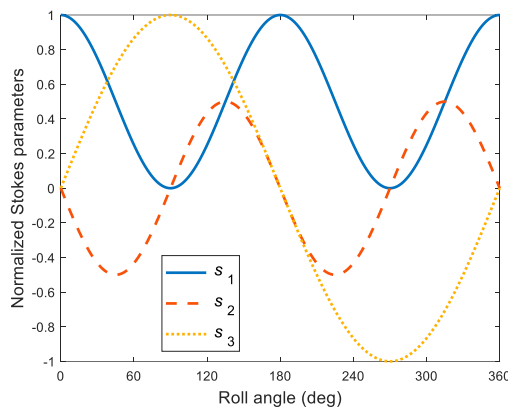


Fig. 2. Simulated Stokes parameters for the roll angle from  $0^\circ$  to  $360^\circ$ .

## Results

In principle, the fast axis angle of the VR can be solved by the measured Stokes vector analytically. However, nonlinear errors existed in the measurement data. The reasons might be the alignment, uniformity of the retardance and manufacturing accuracy of the fast axis angle. In order to get more accurate results, we used a numerical fitting method in the following experiment. First, the VR was rotated by a stepper motor rotation mount with an absolute accuracy of  $0.14^\circ$  and unidirectional repeatability of  $\pm 60 \mu\text{rad}$  as an angle reference. Then a fitting model was established based on the measured data. Finally, the square error function can be written as

$$\chi^2 = \sum_{i=1}^3 (s_i^{\text{Exp}} - s_i^{\text{Fit}}(\phi))^2,$$

where the superscripts Exp and Fit indicate the experimental data and fitted models, respectively. Non-linear optimization method is applied to solve the fast axis angle of the VR. To verify the sensor, the sensing unit (VR) rotates five times from  $0^\circ$  to  $360^\circ$  with a step of  $10^\circ$ . Figure 3 shows the measurement results of the roll-angle sensor and the length of the error bars indicates the standard deviation of the meas-

urements. Within the whole measuring range, the measurement errors are less than  $\pm 0.15^\circ$  and the standard deviations are less than  $0.09^\circ$ . This experiment shows the proposed sensor has high accuracy and high precision. In addition, the construction of the roll-angle sensor is simple and the sensor can be easily set up for existing measurement systems because of the non-contact method and the simple design.

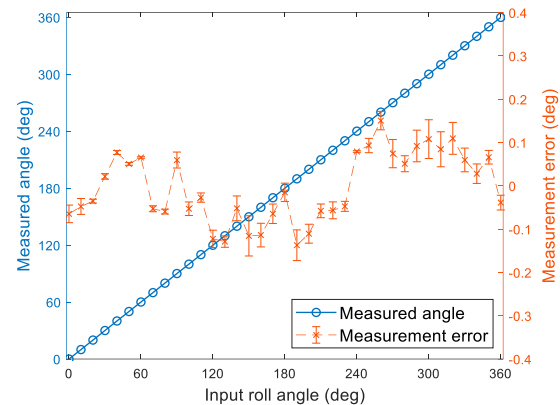


Fig. 3. Results of the measured roll angles compared with the values of a stepper motor rotation mount.

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