Invariance in Measured Quantities across the Sciences

William P. Fisher, Jr1

¹ BEAR Center, Graduate School of Education, University of California, Berkeley, USA, Senior Scientist, Research Institute of Sweden, & Living Capital Metrics LLC, Sausalito CA USA

wfisher@berkeley.edu, wpfisherjr@livingcapitalmetrics.com

Summary:

Basic physical quantities for mass, length, duration, and charge exhibit structural invariances not different in kind from those that also characterize probabilistically measured psychological and social quantities. For over 50 years, the theory and practice of additive conjoint models for measurement introduced in the 1960s have demonstrated that the scope of fundamental measurement is broader than was previously appreciated. This is especially apparent in the correspondences between the various log-interval scales employed in both the natural and the social sciences. These scales are conventionally treated in some fields as ratio scales by choosing convenient exponents, but are commonly expressed as log-odds interval scales in the social sciences. In recent years, a metrological perspective focused on defined quantity values using these kinds of scales has begun to emerge from collaborations of engineers and psychologists. The terms of the shared perspective on measurement concern a basis in modeling lawful regularities, predictive explanatory theories, and quality assured metrological traceability to consensus standards.

Keywords: measurement, modeling, metrology, history, log-interval scales

Interval Scales across the Sciences

Writing in 1986, Narens and Luce [1] say that mathematical models emerging in the 1960s provide "a basis for measuring a number of the basic physical quantities: mass, length, duration, and charge," observing "that much the same structure underlies the measurement of probability." The authors report that use of these models to obtain interval-scalable, fundamental measurements of non-extensive, nonphysical, psychological, and social constructs is "widely accepted." Given that 35 years have passed since that statement was made, the reader may well wonder why higher quality measurement has not yet been more widely achieved.

The class of additive conjoint models being referred to falls under the heading of log-interval scales, which S. S. Stevens [2] added as a fifth entry in his taxonomy of four scale types (nominal, ordinal, interval, and ratio). Narens and Luce note that this kind of scale is used in multiple examples across the sciences (such as decibels, the Richter scale, pH acidity, stellar magnitude, entropy, and information) contrary to common perceptions that it is rare.

Probabilistic models for measurement developed by Rasch [3-4] belong to this class of models [5-7] expressing interval units in log-odds form. Rasch [4] recounted that, when developing

his initial model, "I imagined...that the reading ability of a student could be characterized in a quantitative way--not through a more or less arbitrary grading scale, but by a positive real number defined as regularly as the measurement of length."

These models, their estimation, fit assessment, software applications, implementations, and professionalization were significantly advanced by Wright [7-8], his students and colleagues [9], and Rasch's students [6, 10].

Models and Modeling Take Pride of Place

Nersessian [11] notes that "A significant segment of history and philosophy of science now gives models and modeling pride of place among scientific tools and practices." She and others [12-13] argue that reasoning with model systems takes place within socially distributed cognitive systems as constraint satisfaction processes in which mental and physical models co-evolve. A key point is that the conjoint interactions of mental and physical models do not occur in the isolation of a single person's mind but instead are integrated with cognitive resources embedded in the external shared social environment. These resources take the form of everyday languages' alphabets, dictionaries, phonemes, grammars, etc., as well as the more technically complex standards of scientific languages' unit definitions, mathematical models, instrument calibrations, quality assurance methods, etc.

The similarity or goodness of fit of the model defines the relationship between mental and physical realities in much the same way they do for the relationships between mental and psychosocial models [15]. Successful models support local inferences and insights that cannot be entirely anticipated in research. Models are manipulated by changing their features and trying them out experimentally across contexts, with the aim of assessing their applicability, fit, and usefulness.

Nersessian establishes that scientific modeling does not enjoy any special advantage conferred by a supposed superior tractability of its objects of investigation. She focuses on the ways in which normal everyday cognitive operations are extended in science. Basic processes of analogy embedded in social and technical contexts work much the same way in normal language usage as they do in scientific language usage.

New Metrological Horizons

Nersessian's account of the place of models and modeling in the history of science emphasizes the importance of models that are in principle identifiable: that are structured to have a capacity to locate, describe, and potentially explain repeatably reproducible phenomena. The primary focus of scientific models of this kind is not, then. descriptive, as in statistical modeling, but, rather, prescriptive. This orientation in modeling is important for the lesson learned from history: scientific laws are not discovered via measurement; rather, measurement requires that the laws are already in hand [15]. Thus we have the special significance of the fact that Rasch [3] intentionally structured his probabilistic models to have the same form as Maxwell's treatments of Newton's Second Law. The implications of this capacity to see the same mathematics in geometry, physics, and psychology [16] are increasingly explored in collaborations among metrologists and psychometricians [17-20].

References

- [1] L. Narens, R. D. Luce, Measurement. *Psychological Bulletin*, 99(2), 166-180 (1986); doi: 10.1037/0033-2909.99.2.166
- [2] S.S. Stevens, Mathematics, measurement, and psychophysics. In S. S. Stevens (Ed.), *Handbook* of experimental psychology (pp. 1-49). New York: Wiley (1951)
- [3] G. Rasch, *Probabilistic models*, Chicago: University of Chicago Press, (1980)
- [4] G. Rasch, On specific objectivity. *Danish Year-book of Philosophy*, 14, 58-94 (1977)

- [5] H. E. Brogden, The Rasch model, the law of comparative judgment and additive conjoint measurement. *Psychometrika*, 42, 631-634 (1977); doi: 10.1007/BF02295985
- [6] G. H. Fischer, Applying the principles of specific objectivity and of generalizability to the measurement of change. *Psychometrika*, 52(4), 565-587 (1987); doi: 10.1007/BF02294820
- [7] B. Wright, A history of social science measurement, Educational Measurement: Issues and Practice, 16, 33-52 (1997); doi: 10.1111/j.1745-3992.1997.tb00606.x
- [8] B. D. Wright, Solving measurement problems with the Rasch model. *Journal of Educational Measurement*, 14(2), 97-116 (1977); https://www.jstor.org/stable/1434010
- [9] W. P Fisher, B. D. Wright, (Eds.) Applications of probabilistic conjoint measurement. *International Journal of Educational Research*, 21(6), 557-664 (1994); doi: 10.1016/0883-0355(94)90010-8
- [10] E. B. Andersen, Sufficient statistics and latent trait models. *Psychometrika*, 42(1), 69-81 (1977); doi: 10.1007/BF02293746
- [11] N. J. Nersessian, Model-based reasoning in distributed cognitive systems. *Philosophy of Science*, 73, 699-709 (2006); https://doi.org/10.1086/518771
- [12] N. J. Nersessian, Creating scientific concepts. Cambridge, Massachusetts: MIT Press (2008)
- [13] B. Latour, *Science in action*. New York: Harvard University Press (1987)
- [14] T. S. Kuhn, The function of measurement in modern physical science. Isis, 52(168), 161-193 (1961); doi: 10.1086/349468
- [15] A. J. Stenner, W. P. Fisher, M. H. Stone, D. S. Burdick, Causal Rasch models. Frontiers in Psychology: Quantitative Psychology and Measurement, 4(536), 1-14 (2013); doi: 10.3389/fpsyg.2013.00536
- [16] W. P. Fisher, A. J. Stenner, On the potential for improved measurement in the human and social sciences. In Q. Zhang & H. Yang (Eds.), Pacific Rim Objective Measurement Symposium 2012 Proceedings (pp. 1-11). Berlin: Springer (2013); doi: 10.1007/978-3-642-37592-7
- [17] L. Mari, M. Wilson, An introduction to the Rasch measurement approach for metrologists, *Measurement*, 51, 315-327 (2014); doi: 10.1016/j.measurement.2014.02.014
- [18] L. Pendrill, Quality assured measurement. Cham: Springer (2019)
- [19] L. Pendrill, W. P. Fisher, Counting and quantification. *Measurement*, 71, 46-55 (2015); doi: 10.1016/j.measurement.2015.04.010
- [20] L. Mari, M. Wilson, A. Maul, Measurement across the sciences. Cham: Springer (2021)