

$$f_r = (c_1 + c_2\rho + c_3\sqrt{\rho\eta/f_r})^{-\frac{1}{2}}, \quad (1a)$$

$$Q = \frac{1}{f_r}(c_4 + c_5\eta + c_6\sqrt{\rho\eta f_r})^{-1}, \quad (1b)$$

We propose an alternative model shown in eq. (2), which allows for an arbitrary degree of approximation, not limited to second order.

$$\underline{G}(f_r, Q) = \rho \sum_{i=0}^N \underline{c}_i \delta^i \quad (2a)$$

$$\delta = \sqrt{\frac{\eta}{\pi f_r \rho}} \quad (2b)$$

We found an elegant new form where \underline{G} represents a complex-valued function of the resonance parameters and \underline{c}_i are complex-valued instrument parameters obtained by adjustment measurements using test fluids. δ is the characteristic decay length of plane shear waves. The inversion of the model is straight-forward and its complexity remains similar, when the order of approximation is extended to any arbitrary truncation N of the Taylor series. Furthermore, new physical insights could be obtained using basic dimensional analysis [3]. It could be shown that this model is applicable for mechanical resonators of any shape, given that effects due to fluid compressibility and nonlinearity can be rendered negligible by design. The QTF sensor implemented in the fluidFOX in Fig. 1 is such a candidate.

Tab. 1: Table of the nominal values of the fluids measured using the fluidFOX. Deviations for viscosity are shown in Fig. 2

fluid #	Fluid	Temp. (°C)	Density (kg/m ³)	Dyn. visc. (mPas)
1	N2	50	740.1	1.2760
2	* N2	40	747.2	1.5060
3	N2	25	757.8	1.9930
4	N2	20	761.4	2.2120
5	N7	50	781.1	4.4050
6	N7	40	787.7	5.7680
7	N14	50	793.3	8.2420
8	N7	25	797.5	9.2720
9	N7	20	800.8	11.110
10	N14	40	799.7	11.290
11	N26	50	801.9	14.350
12	N14	25	809.3	19.670
13	* N26	40	808.2	20.470
14	N44	50	808.7	23.410
15	N14	20	812.5	24.320
16	N44	40	814.8	34.540
17	N26	25	817.6	38.050
18	N26	20	820.8	48.140
19	* N44	25	824.1	68.120
20	N140	50	819.1	71.950
21	N44	20	827.2	87.860
22	N140	40	825.1	112.20
23	* N140	25	834.1	242.90

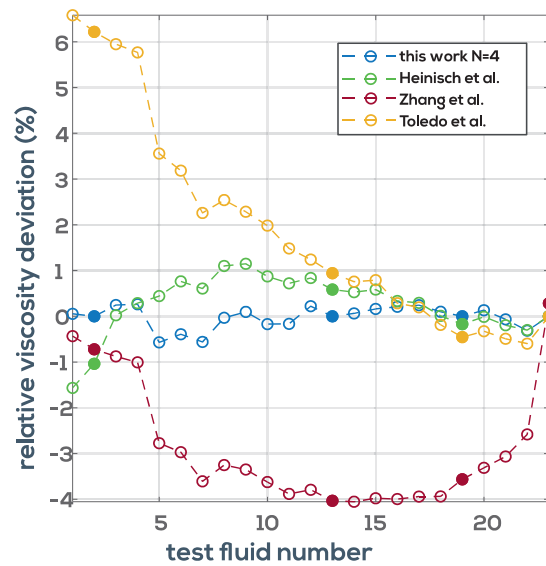


Fig. 2. Deviations between different fluid models. The viscosities of the fluid range from 1.27mPas to 242.9mPas. Fluids indicated by a filled marker correspond to fluids marked with * in Tab. 1 and were used to adjust the model parameters c_i (see [5] for details).

We applied various models to the raw data available from the fluidFOX. Fig. 2 shows viscosity deviations where the new model used a truncation of $N=4$. The used respective NIST traceable reference fluids are listed in Tab. 1. The fluid marked with * were used to adjust the model parameters.

Conclusions

A new fluid model for simultaneous measuring of viscosity and density was established showing distinct advances over existing models.

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