Ultrasonic Sensor Viscosity of Mineral Oil for Built-in Pipe Applications

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Abstract
This paper presents an empirical method for measuring the viscosity of mineral oil. In a built-in pipeline application, conventional ultrasonic methods using shear reflectance or rheological and acoustical phenomena may fail due to attenuated shear wave propagation and an unpredictable spreading loss by protective housings, and comparable main flows. The empirical method utilizing longitudinal waves eliminates the unknown spreading loss from attenuation measurements on the object fluid by removing the normalized spreading loss per focal length with the measurement of a reference fluid of a known acoustic absorption coefficient. The ultrasonic attenuation of fresh water as the reference fluid and mineral oil as the object fluid were measured along with the sound speed and effective frequency. The empirical equation for the spreading loss in the reference fluid is determined by high-order polynomial fitting. To estimate the shear viscosity of the mineral oil, a linear fit is applied to the total loss difference between the two fluids, whose slope (the absorption coefficient) is combined with an assumed shear-to-volume viscosity relation. The empirical method predicted the viscosities of two types of the mineral oil with a maximum statistical uncertainty of 8.8% and a maximum systematic error of 12.5% compared to directly measured viscosity using a glass-type viscometer. The validity of this method was examined by comparison with the results from theoretical far-field spreading.

Introduction
The purpose of the empirical approach in this study is to extract the shear viscosity of fluids from the volume viscosity in the acoustic absorption. The volume viscosity was theoretically derived by Stokes [1] in addition to the shear viscosity, but its existence could not be attested until high frequency ultrasonic methods were developed. The ultrasonic measurements in fluids could demonstrate the additional absorption by the volume viscosity relative to the predicted absorption owing to shear viscosity alone. The volume viscosity for unassociated liquids such as Benzene and Toluene is mostly caused by relaxation phenomena [2], in which energy redistribution occurs among the internal degrees of freedom of molecules by transmitting acoustic waves to the fluids. For associated liquids with relatively strong intermolecular bonding, structural relaxation is dominant in the volume viscosity. According to Eyring's hole theory [3], the structural relaxation is explained as a flow of molecules into lattice position in fluids under acoustic pressure. The flow takes a finite time to closely pack and rearrange the molecules, which results in the phase change of the acoustic pressure and causes the additional absorption. Within a frequency range where dispersion is negligible, the absorption coefficient caused by the shear viscosity η_S and volume viscosity η_V is expressed as [4]

$$\alpha = \frac{2\pi^2 f^2}{\rho_0 c_f^{\ 3}}\left(\frac{4}{3}\eta_S + \eta_V\right) \quad \text{(neper/m)} \qquad (1)$$

where f is the excitation frequency, ρ_0 the density of the fluid, and c_f the sound speed in the fluid. To calculate the shear viscosity from the measured absorption coefficient, it is convenient to define a viscosity-relation constant as

$$K_v = \frac{\eta_A}{\eta_S} = \frac{4}{3} + \frac{\eta_V}{\eta_S} \qquad (2)$$

where η_A is designated as the acoustic viscosity. Litovitz and Davis [5] reported the ratio η_V/η_S in several types of liquids, which can then be used to calculate K_v values. In monatomic liquids such as mercury, K_v was found to be 2.53, and 2.33 for the non-associated organic liquid, CS_2. In polymeric liquids such as Polyisobutylene, increasing molecular weight affected the shear and volume viscosities in the same manner. Among hydrogen-bond liquids the temperature dependence of the two viscosities was found to be very similar, and the corresponding K_v values changed insignificantly. The approximate value of K_v for a sample of hydrocarbon oil was found as 2.667 by the study of structural relaxation [6]. The structural relaxation behavior of hydrocarbon oil was observed as very similar to that of the hydrogen-bond liquids. The mineral oil used in this study was classified as hydrocarbon oil and assumed to share the same value of K_v. The variations of K_v, which were observed in other types of oil products [7] in a wide range of temperature and frequency, were neglected as assuming that these parameters are constant during the measurement.

For the physical model of the acoustic attenuation, we begin with an assumption that transducers with a diameter of D are theoretical baffled-piston sources immersed in a viscous fluid. The acoustic pressure P_{in} generated by the transmitting transducer is observed as P_{out} as propagating a distance d in the far-field. The relation between the two acoustic pressures is expressed

$$\frac{P_{out}}{P_{in}} = \left(\frac{\pi D^2}{4d\lambda}\right) \mathrm{e}^{-\alpha d} \tag{3}$$

where α is the acoustic absorption coefficient (neper/m) and λ the wavelength in the fluid. The parenthesis in Eq. 3 represents the beam spreading, and the exponential term is the attenuation by the acoustic absorption. It is inferred that the theoretical beam spreading is a function of three geometric parameters: the transducer diameter D, the propagating distance d, and the wavelength λ. The acoustic viscosity used in Eq. 2 is expressed as

$$\eta_A = \frac{\rho_0 {c_f}^3}{40\pi^2 f^2 (\log_{10} e)} \alpha' \quad \text{(Pa·s).} \tag{4}$$

Herein the absorption coefficient is given in dB/m and is noted α'. Equations 2 and 4 give the expression of the shear viscosity as

$$\eta_S = \frac{\alpha' \rho_0 {c_f}^3}{171.45 K_v f^2} \quad \text{(Pa·s).} \tag{5}$$

This is converted into kinematic viscosity in centistokes (cSt) as

$$\nu = \frac{\eta}{\rho_0} \times 10^6 = \frac{5832.5 \alpha' {c_f}^3}{K_v f^2} \quad \text{(cSt).} \tag{6}$$

This equation shows that the estimation of the shear viscosity of the fluid calls for the properly defined viscosity-relation constant K_v, and the accurately measured absorption coefficient α', sound speed c_f, and exciting frequency f.

Absorption Coefficient and Viscosity

The acoustic absorption coefficient is measured by subtracting the spreading loss in a reference fluid from the measured total loss in a fluid of unknown viscosity. Figure 1 shows relevant acoustic paths for a pitch-and-catch configuration. The transmitting transducer converts an electric signal input V_{in} into mechanical waves which pass through a housing. The housing emits an acoustic pressure P_{in}, which travels a distance d in the viscous fluid as undergoing the acoustic absorption and the beam spreading. The housing of the receiving transducer receives the acoustic pressure P_{out}, and finally the output electric signal V_{out} is generated from the receiving transducer. The receiving transducer is assumed to be placed in the far-field (far beyond the acoustic focal length given by $N_d=D^2/4\lambda$). The two transducers are assumed to be reciprocal and have identical sensitivities. The coherent interference of the signal from acoustic paths other than the fluid is neglected. Minor electrical and mechanical losses may occur as the signal propagates, but they are assumed as linearly proportional to the original signal amplitude and unrelated to the geometry and acoustic absorption. These assumptions allow us to disregard losses except for the spreading and absorption losses as an offset in decibels.

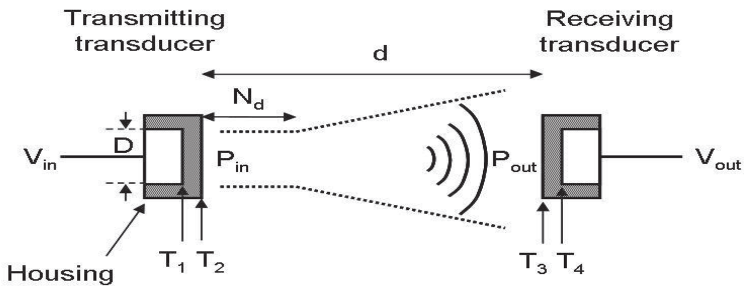

Figure 1. Acoustic paths of the signal.

The voltage-pressure relations are simply expressed as:

$$\begin{aligned} P_{in} &= T_1 T_2 V_{in} \sqrt{n} \\ V_{out} &= T_3 T_4 P_{out} \sqrt{n} \\ \frac{P_{out}}{P_{in}} &= \frac{1}{T_1 T_2 T_3 T_4} \frac{1}{n} \frac{V_{out}}{V_{in}} \end{aligned} \tag{7}$$

where n is the power efficiency of the transducer defined as $2k_T^2/\pi$, and k_T^2 is the effective piezoelectric coupling coefficient [8]. T_1 to T_4 are corresponding transmission coefficients to individual interfaces as shown in Fig. 1. Practically, only voltage amplitudes can be measured (V_{in} and V_{out}) instead of the acoustic pressures. Even though metallic housings exist on the transducers, a baffled-piston assumption is used in Eq. 3; this assumption will be removed later. The ratio of the two measured voltages is combined with Eq. 3 and expressed as

$$\frac{V_{out}}{V_{in}} = n\, T_1 T_2 T_3 T_4 \frac{\pi D^2}{4d\lambda} \mathrm{e}^{-\alpha d} = n\, T_{tot} \frac{\pi D^2}{4d\lambda} \mathrm{e}^{-\alpha d} \tag{8}$$

where T_{tot} is the multiplication of all the transmission coefficients. The inverse of voltage ratio is converted into the total loss H_{tot} in decibels and expressed as an algebraic sum of losses:

$$H_{tot} = H_{trans} + H_{spread} + H_{absorp} + H_{eff} \quad \text{(dB)}, \tag{9}$$

where H_{spread}, H_{absorp}, H_{trans}, and H_{eff} are the losses for beam spreading, absorption, transmission through the housings, and power efficiency in the transducers, respectively. These are defined from Eqs. 3, 7 and 8 as Eq. 10:

$$\begin{aligned} H_{tot} &= 20 \log_{10} \left(\frac{V_{in}}{V_{out}} \right) \\ H_{trans} &= 20 \log_{10} \left(\frac{1}{T_{tot}} \right) \\ H_{spread} &= 20 \log_{10} \left(\frac{4d\lambda}{\pi D^2} \right) \\ H_{absorp} &= 20 \log_{10} \left(\mathrm{e}^{\alpha d} \right) = (20\alpha \log_{10} e) d = \alpha' d \\ H_{eff} &= 20 \log_{10} \left(\frac{1}{n} \right) = 20 \log_{10} \left(\frac{\pi}{2k_T{}^2} \right) \end{aligned} \tag{10}$$

Taking into account Eqs. 9 and 10, the difference between the total loss and spreading loss can be written as the linear relation:

$$H_{tot} - H_{spread} = \alpha' d + H_{trans} + H_{eff} \quad \text{(dB)}, \qquad (11)$$

which represents a difference between the total and spreading losses. For the transmission and efficiency losses are not a function of the acoustic absorption and geometry, Eq. 11 becomes a linear function of *d*. The term associated with the absorption coefficient in Eq. 11 can be combined with Eq. 6 and expressed as

$$\alpha' d = \frac{K_v \nu}{5832.5} \frac{f^2}{{c_f}^3} d = \frac{K_v \nu}{5832.5 f \lambda^2} \delta \quad \text{(dB)}, \qquad (12)$$

where *δ* is the normalized distance defined as *d/λ*. The wavelength *λ* can be calculated from the measured sound speed and the effective frequency *f*. Note that the effective frequency may differ from the exciting frequency of the electric signal. In a broadband signal, a frequency shift occurs because of the absorption in the viscous fluid and results in a variation of the wavelength. The wavelength variation is taken into account to calculate the focal length N_d, which is utilized for the normalization of the spreading loss Eq. 10 as:

$$H_{spread} = 20\log_{10}\left\{\frac{d}{\pi N_d}\right\} = -20\log_{10}\pi + 20\log_{10}(\zeta) \quad \text{(dB)}, \qquad (13)$$

where *ζ* is the normalized distance defined as d/N_d (also referred to as the Seki Parameter [9]). In a real installation on the pipeline, transducers are inevitably equipped with metallic housings, and the receiving transducer may not be placed in the far-field due to the limited pipe diameter. Under these restrictions, the far-field spreading loss expression in Eq. 13 is not valid any more. Instead, it can be expressed as a sum of two unknown terms: distance-dependent function F(*ζ*) and a constant determined by the type of fluids as

$$H_{spread} = F(\zeta) + G \quad \text{(dB)}, \qquad (14)$$

Herein the use of the normalized variables allows the isolation of the spreading losses among different types of fluid. From the measured total loss H_{tot} of a reference fluid with a known absorption coefficient, we can express the H_{spread} as a function of the Seki parameter, which can be applied to any kind of fluid to cancel out the distance-dependence of the spreading loss. For the oil viscosity measurement, fresh water is used as the reference fluid and the total losses $H_{tot(oil)}$ and $H_{tot(water)}$ of the two fluids are measured as well. Since the absorption coefficient of the reference fluid is known, its total loss excluding the absorption loss is defined as $H'_{tot(water)}$. The relation of these losses is manipulated as a linear equation:

$$H_{tot(oil)} - H'_{tot(water)} - \left\{H_{trans(oil)} - H_{trans(water)}\right\} = \frac{K_v \nu}{5832.5 f \lambda^2}\delta + \left(G_{oil} - G_{water}\right) \quad \text{(dB)}.(15)$$

The efficiency and transmission loss can be considered, but an offset is left in the right-hand side of Eq. 15 due to the differences in acoustic impedances and focal length in each medium. The shear viscosity *v* in the oil product can be estimated by the following procedure:

- Measure the total loss $H_{tot(water)}$ of a fluid with known viscosity as a function of the distance.
- Obtain $H'_{tot(water)}$ in fresh water by numerically fitting with a high-order polynomial.
- Measure the total loss $H_{tot(oil)}$ by the same procedure of measuring $H_{tot(water)}$.
- Calculate H_{trans} in each fluid.

- Obtain loss difference between the two fluids by subtracting the expression $H'_{tot(water)}$ from $H_{tot(oil)}$.
- Apply the linear fitting to the right-hand side of Eq. 15.
- Calculate the shear viscosity based on the assumed K_v value and the slope of the linear equation.
- Speed and effective frequency should be measured to obtain the wavelength in the fluid.

Estimation of the Viscosity Using the Empirical Method
The fourth-order polynomial $y=A_4\zeta^4+A_3\zeta^3+\cdots+A_0$ was utilized to obtain the empirical equation for the spreading loss of the fresh water (see Fig. 2), where ζ represented the Seki parameter. The measurement range included both the near-field (ζ<4) and far-field (ζ>4). The coefficients were found as follows: Range 1) A_4=0.0001839, A_3=-0.0038382, A_2=-0.003838, A_1=1.349, and A_0=29.97; Range 2) A_4=0.009092, A_3=-0.1375, A_2=0.6792, A_1=-0.05022, and A_0=30.78. The corresponding values of these equations were subtracted from the total loss in the mineral oil to obtain the total loss difference. The transmission loss differences, ($H_{tran(water)}$-$H_{tran(oil)}$) were calculated as 2.00 dB for Drakeol 5 and 1.26 dB for Drakeol 600 and used in Eq. 15. A linear fit applied to the measured data resulted in the linear equation $y=ax+b$ and correlation factor R. The slope a was used to calculate the shear viscosity in Eq. 15 by

$$\nu = \frac{5832.5 f \lambda^2}{K_v} a \quad \text{(cSt)}, \qquad (16)$$

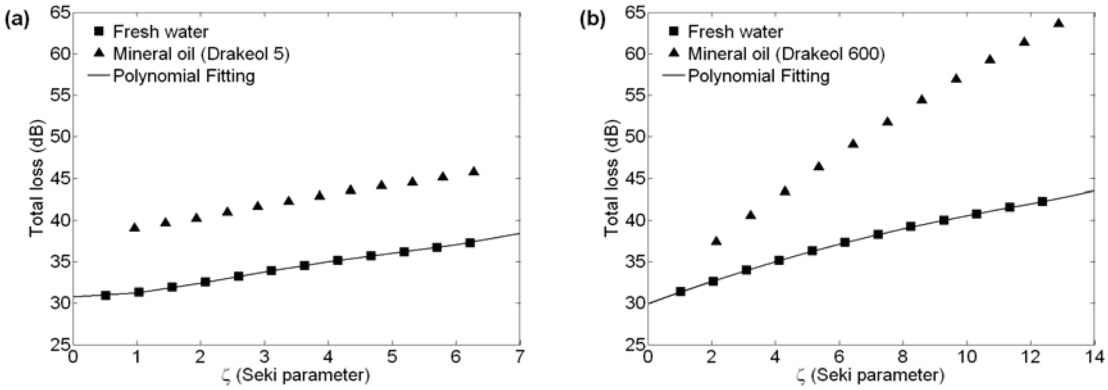

Figure 2. Losses versus normalized distance in fresh water and mineral oil. (a) Drakeol 5 with Range 2 (b) Drakeol 600 with Range 1. The empirical equation was fitted to total loss in water using fourth-order polynomial.

and R for the uncertainty in Eq. 16. The offset represented by b remained non-zero due to unknown external losses, which may come from transducer efficiency of different liquid-loading, transducer misalignment, or spreading loss contributed by liquids properties such as wavelength and propagation velocity. Inevitable electrical leakage to the liquids also resulted in the offset. However, the offset was neglected at this stage because it might not be distance-dependent. Only the slope was utilized for the viscosity estimation. Parameters for Eq. 16 were selected from Table 1 and K_v is assumed as 2.667 for both hydrocarbon oil samples (adopted from [6]). The resulting viscosities estimated by the empirical method are shown in Table 2. The empirically measured shear viscosities of the mineral oils were compared to directly measured viscosity by the glass-type viscometers (Cannon-Frenske routine type 150 for Drakeol 5 and type 350 for Drakeol 600, Cannon Instrument, State College, PA). By interpolation, the viscosities were estimated as 12.91 cSt for Drakeol 5 at 23.3 °C and 310.4 cSt for Drakeol 600 at 22.6 °C, respectively. These values are comparable to the empirical results, 12.50 cSt (3.2%) for Drakeol 5 and 271.7 cSt (12.5%), where the values in the parentheses represent a deviation or systematic error from the directly measured results.

Table 1. Sound speed and effective wavelength

	Fresh water	Drakeol 5	Drakeol 600
Sound speed (m/s)	1498	1397	1477
Sound speed Uncertainty (%)	0.16	0.033	0.034
Effective frequency (MHz)	0.968	0.964	0.918
Wavelength (mm)	1.548	1.449	1.610

Table 2. Viscosity measurement results

Mineral oil type	Temp. (°C)	v (empirical) (cSt)	v (reference) (cSt)	Statistical error (%)	Systematic error (%)
Drakeol 5	23.3	12.50	12.91	8.8	3.2
Drakeol 600	22.6	271.7	310.4	1.2	12.5

Conclusion

An empirical method and preliminary results were introduced to estimate the shear viscosity of the oil by means of the ultrasonic attenuation of longitudinal waves. In spite of the limited number of oil samples used in this experiment, the method discussed in this study of a viscosity measurement utilizing ultrasound as a non-invasive method for potential in-line pipeline measurements at its initial stage would improve the current state of art in this industry. Application of the method resulted in an estimate of the shear viscosities of two types of the mineral oil with a maximum statistical uncertainty of 8.8% and a maximum systematic error of 12.5% compared to the directly measured viscosity. The fact that the statistical uncertainty in Drakeol 600 is less than the calculated error suggests that the assumed value of the constant K_v, which accounts for the relationship between shear and volume viscosity, may not be quite accurate. This relationship may vary with the chemical structure of the fluid, temperature and frequency, but the relevant database for all these various conditions is still undetermined. Therefore, a new study is recommended for future work to establish a database to include the various types of liquids (heavy and light oils) used in industry for sets of K_v under a wide range of frequencies and temperatures [10].

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