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Co-Calibration in Distributed Homogeneous Sensor Networks

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Summary:
A co-calibration method suited only for (strictly) homogeneous sensor networks is applied to distributed homogeneous sensor networks. This is achieved by relying on spatial and temporal interpolation models to provide virtual reference measurement points at the location of the device under test. The interpolation method is evaluated in a simulation of an existing real-world use case dealing with room-temperature monitoring using distributed sensors.

Keywords: co-calibration, distributed sensor networks, spatio-temporal interpolation

Background and Motivation

Current work by the authors is focused on uncertainty-aware co-calibration of local homogeneous sensors, as well as spatial interpolation using machine-learning approaches [1].

Requiring sensors to be quasi non-distributed is a strong assumption that greatly limits the potential applicability of co-calibration in practical scenarios. To overcome this limitation, it is shown that distributed sensors can provide virtual reference values by augmenting interpolation models with GUM uncertainty evaluation [2]. Moreover, multiple interpolation models can be evaluated in parallel at a given spatio-temporal point, which can then be robustly combined via sensor fusion into a single virtual reference measurement. The approach is implemented inside a proof-of-concept simulation environment representing temperature sensors distributed inside a room.

Idea Outline

A co-calibration is similar to a calibration according to the VIM [3], but is carried out under non-ideal conditions, e.g., inside an industrial process. The result of a co-calibration method are traceable estimates of parameters characterizing a sensor's transfer behavior (e.g., linear-affine). It is of interest to provide a method for co-calibration of homogeneous sensors that is capable to also operate on spatially distributed and temporally non-synchronous input measurement.

Such a co-calibration involves spatial and temporal interpolation with a use case specific model. The interpolation model has the available measurements by reference devices as inputs and provides virtual reference measurements at the spatio-temporal positions required for the co-calibration of the device under test. The interpolation model can thus be interpreted as a means of performing virtual measurements.

Setting

Consider the case of N distributed calibrated reference sensors monitoring a quantity $\phi(\vec{x},t)$. The n-th $(n = 1, \ldots, N)$ reference sensor is spatially located at $\vec{x}_n$ and provides estimates $\hat{\phi}_n\left(\vec{x}_n, t_{n,j}\right)$ of ϕ at discrete points in time $t_{n,j}$. Sensor readings are not expected to be synchronized but are assumed to refer to the same time base. Locations and timestamps could also have associated uncertainty. The co-calibration expects M time-series of length K as reference measurement $\hat{\phi}^*_m(\vec{x}_{dut}, t_k)$ with uncertainty $u\left(\hat{\phi}^*_m(\circ)\right)$ at the position $\vec{x}_{dut}$ of the device under test as well as its indicated values $y(t_k)$ at the same consecutive time-points t_k with $k = 1, \ldots, K$. From the input, the co-calibration iteratively estimates the parameters $\left(a, b, \sigma_y\right)$ of a linear-affine transfer behavior with gain a, offset b and noise $\varepsilon \sim \mathcal{N}\left(0, \sigma_y^2\right)$ using Bayesian updates. The sensor's transfer model is assumed to be given by

$$y(t_k) = a \cdot \phi(\vec{x}_{dut}, t_k) + b + \varepsilon_i\,.$$

As it can in general not be expected that $\vec{x}_n = \vec{x}_{dut}$ or $t_{n,j} = t_k$ for any n, j or k, the available measurement data from the reference sensors does not match the input requirements of the co-calibration routine. The task to be solved is to obtain an estimate $\hat{\phi}^*_m(\vec{x}_{dut}, t_k)$ (and corresponding uncertainty) of ϕ at location $\vec{x}_{dut}$ and timestamps t_k by means of interpolation.

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Methods

A method to carry out uncertainty-aware interpolation in space using Kriging methods was recently investigated [1]. This method can also be extended to cover the asynchronous case. The time-signal of each reference sensor is first interpolated onto the same synchronized timestamps t_k, e.g., using [4]. A spatial Kriging interpolation is then applied to each of these timestamps t_k, resulting in functions over time that can be evaluated at the position of the device under test $\vec{x}_{dut}$.

Here, another approach is presented that does not interpolate independently in time and space, but simultaneously using an adjusted 4D nearest neighbor regression. In a rather low-informative approach, the quantity field $\phi(\vec{x}, t)$ over some finite time interval is approximated in a local multi-dimensional first order approach.

$$\tilde{\phi}(\vec{x}_0, t_0) = \vec{\nabla}_s \cdot \left(\begin{bmatrix} \vec{x}_0 \\ t_0 \end{bmatrix} - \begin{bmatrix} \vec{x}_s \\ t_s \end{bmatrix} \right) + \phi_s$$

Where the gradient $\vec{\nabla}_s$, offset ϕ_s and point of support $\vec{x}_s, t_s$ are obtained from available reference data of the nearest L measurement points using heuristics and a least-squares fit. Neighborhood is defined by the “p=2”-norm of a combined spatio-temporal vector $\begin{bmatrix} \vec{x} \\ t \end{bmatrix}$. The neighborhood of $(\vec{x}_0, t_0)$ is denoted as $\mathcal{N}_L(\vec{x}_0, t_0)$ and the interpolation model given by:

$$\phi_s = \mathrm{median}\left(\hat{\phi}_n(\vec{x}_n, t_{n,j}) \; : (n, j) \in \mathcal{N}_L(\vec{x}_0, t_0) \right),$$

$$\vec{x}_s = \mathrm{mean}(\{\vec{x}_n : \; (n, j) \in \mathcal{N}_L(\vec{x}_0, t_0)\}),$$

$$t_s = \mathrm{mean}(\{t_{n,j} : \; (n, j) \in \mathcal{N}_L(\vec{x}_0, t_0)\}),$$

$$\vec{\nabla}_s = \underset{\vec{\nabla}}{\mathrm{argmin}} \sum_{(n,j) \in \mathcal{N}_L(\vec{x}, t))} \left\| \hat{\phi}_n(\vec{x}_n, t_{n,j}) - \tilde{\phi}(\vec{x}_n, t_{n,j}) \right\|.$$

The parameters $\vec{\nabla}_s, \phi_s$ have associated uncertainties. The uncertainty $u(\phi_s)$ is given by those of the median element(s). The covariance matrix $U(\vec{\nabla}_s)$ is obtained from a (repeated) Monte-Carlo-evaluation of the minimization routine. Applying the “law of propagation of uncertainty” [2] then yields for the uncertainty of $\tilde{\phi}(\vec{x}, t)$:

$$u\left(\tilde{\phi}(\vec{x}_0, t_0)\right) = \sqrt{\left(\begin{bmatrix} \vec{x}_0 \\ t_0 \end{bmatrix} - \begin{bmatrix} \vec{x}_s \\ t_s \end{bmatrix} \right) U(\vec{\nabla}_s) \left(\begin{bmatrix} \vec{x}_0 \\ t_0 \end{bmatrix} - \begin{bmatrix} \vec{x}_s \\ t_s \end{bmatrix} \right)^T + u(\phi_s)^2}.$$

Interpolation schemes based on different (potentially overlapping) neighborhoods could be used in parallel. A consequence of the chosen model for ϕ is an increased uncertainty for the interpolated value at points further away from the point of support.

Application

The outcome of applying the proposed interpolation method to a simulated temperature room use case is shown in Figures 1 and 2. The true field is given by the following equation (with spatially dependent amplitude, offset and phase):

$$\phi(\vec{x}, t) = 2|\vec{x}| + |\vec{x}| * \sin(t + |\vec{x}|)$$

Depending on the distribution of the sensors (black circles in Figure 2), closer reference measurements result in better estimates of the field ϕ. The GUM-propagated uncertainty of the second interpolation model supports this observation by providing higher uncertainty values in regions (4D) that are further away from existing reference measurements.

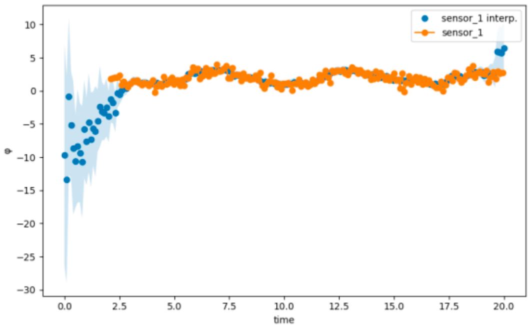

Figure 1: Interp. at position of an existing sensor.

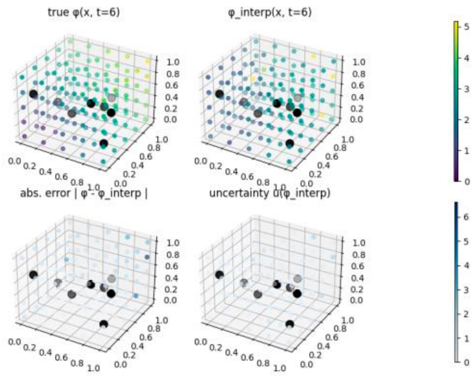

Figure 2: Interpolation at t=6 over the obs. volume. Sensor positions are drawn as black circles.

Conclusion and Outlook

Two uncertainty-aware spatio-temporal interpolation methods are proposed. One is presented in detail to use non-synchronized spatially distributed sensor network data as input for a homogeneous co-calibration method. Both approaches propagate the uncertainty into the interpolated value, but do not weight measurement data based on uncertainty.

It is of interest to further adapt, compare and explore existing and new interpolation schemes.

References

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