DOI 10.5162/SMSI2023/B7.2

# Monolithic Guiding Mechanism and Adjustment Devices for an Electrostatic Force Balance at LNE

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**Summary:**

A measurement apparatus that uses an electrostatic force generator is being developed at LNE, toward the realization of small masses and forces in the International System of Units (SI). Two of the main parts are designed, i.e. a parallelogram balance mechanism, and two capacitance actuators. The balance mechanism is fully monolithic as well as the two adjustment systems of the electrostatic actuators. Special attention has been paid to the overall symmetry of the system and to cost efficiency.

**Keywords:** Balance, electrostatic force, measurement, mass.

## Introduction

The principle of the electrostatic balance method for measuring small forces (range 1 mN to 1 nN) and thus small masses (range 100 mg to 100 µg) proposed by [1], [2], can be used to realize mass at the 1 mg level with a standard uncertainty below 10 ppm [3].

When a voltage $V$ is applied on a capacitor, an electrostatic force $F$ is generated:

$$F = -\frac{1}{2}\frac{\mathrm{d}C}{\mathrm{d}z}V^2 \qquad (1)$$

where $\mathrm{d}C/\mathrm{d}z$ is the spatial gradient of the capacitance. This force can be used to balance the gravity force acting on a mass artefact $m$ by using a balance mechanism. It allows to compare forces along its compliant axis: to achieve that, one of the electrode of the capacitor is affixed on the balance mechanism, whereas the second one is set immobile in laboratory referential. In a first approximation, the mass value can be determined as:

$$m = \frac{1}{2\,g}\frac{\mathrm{d}C}{\mathrm{d}z}\left(V_{\mathrm{off}}^2 - V_{\mathrm{on}}^2\right) \qquad (2)$$

where $g$ is the local acceleration of gravity and $V_{\mathrm{off}}$ (resp. $V_{\mathrm{on}}$) the voltage needed to equilibrate at the $z_0$ altitude the balance mechanism without the mass sample (resp. with the mass sample).

Determining $\mathrm{d}C/\mathrm{d}z$ requires translating one of the electrode of the capacitance (by using the balance mechanism as a guiding stage, and usually by using a second capacitance as an actuator) and measuring $C$ at different $z$: the value $\mathrm{d}C/\mathrm{d}z$ at $z_0$ is then usually obtained by a polynomial fit [4].

## Mechanical Design of Balance Mechanism

Different systems can be used to build a highly compliant mechanism, with linear or quasi-linear movement and parallel motion linkage, Peaucellier–Lipkin linkage… have been investigated. At last, a simple parallelogram has a bunch of desirable feature, as noted by [5] and is probably the easier to machine, has only 4 axis of rotation (see figure 1) and deviates only of 5 µm from a linear trajectory (with the dimensions describes lower) on a 1 mm vertical travel.

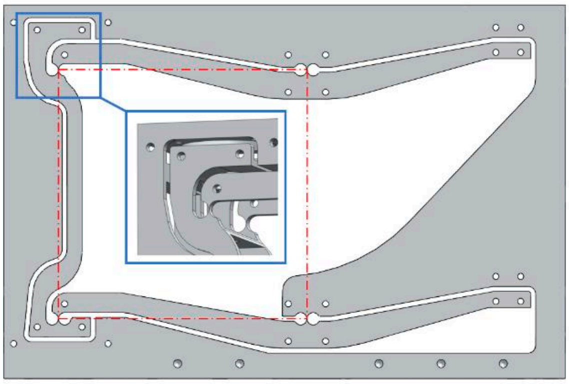

*Fig. 1. CAD lateral view of the balance parallelogram mechanism. On the left, the vertical segment (pan) is the one on which gravity and electrostatic force will be compared. The horizontal arms (swings) are the actual guiding stage and the right extent of the swing are used to have a naturally equilibrated system, when no electrode is fixed on it. The red dashed line indicates the parallelogram position at weighing point (length 100 mm). The inset in the blue square shows a closer view of one hinge.*

Flexures hinges have several key advantages over other solutions, and monolithic design, with for example a wire electrical discharge machining (WEDM), yields major benefits [5], and notably ensure the alignment of this delicate mechanism [6].

The chosen hinges are circular ($r = 2.5$ mm) with a thickness $t$ of 40 µm. In order to improve transverse stiffness without compromise on the torsional stiffness, each hinge is indeed a comprised of two single hinges of width $w$ 3 mm separated by 34 mm. Being electro-machined in aluminium EN AW-7075 T651, each half hinge will have a flexure stiffness of [7]:

$$\kappa_{\text{half hinge}} = \frac{2\,E\,w}{9\,\pi}\sqrt{\frac{t^5}{r}} = 3.1 \cdot 10^{-3}\ \mathrm{N\,m\,rad^{-1}} \quad (3)$$

with $E = 72\ \mathrm{GPa}$ the Young's modulus. Of course, its stiffness is slightly modified once the hinge is loaded.

Regarding the dimensions of the whole system, following [5], we have chosen a length $b = 100\ \mathrm{mm}$ for each of the arms of the apparatus. Thus the vertical stiffness of the balance mechanism, comprised of its eight half hinges will be [8]:

$$K = \frac{\kappa}{b^2} = 2.5\ \mathrm{N\,m^{-1}} \quad (4)$$

The balance mechanism is ready to welcome a spring system in the near future to lower this stiffness [8].

**Capacitance and Adjustment Mechanism**

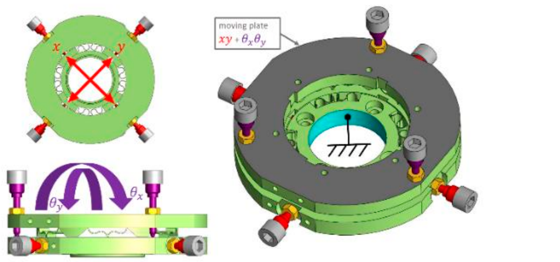

*Fig. 2. CAD lateral view of the 5 dof ($z$ not shown on figure) monolithic adjustment system which will be used to tune the relative position of the outer electrode with respect to the inner one.*

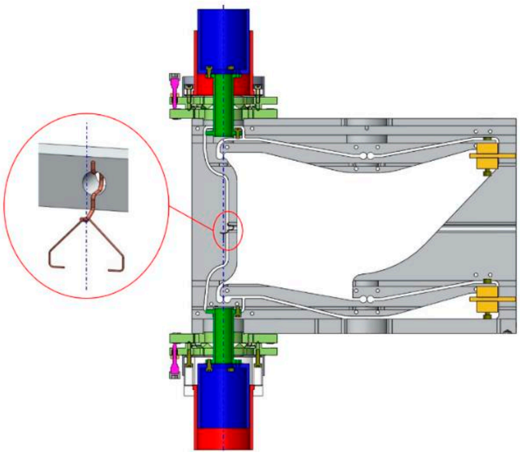

*Fig. 3. CAD sectional general view of the balance parallelogram mechanism, capacitance and adjustment system, mass pan (red circle) and equilibrium mass (orange at left). The close up shows a possible configuration for the mass pan.*

In order to weight 100 mg with a voltage of around 1000 V, the $\mathrm{d}C/\mathrm{d}z$ should be equal to $2\ \mathrm{nF/m}$ at the weighing point. The electrodes are then chosen circular, the capacitance gradient is then constant and depending only (at first approximation) of the ratio of the radii of the electrodes $R_1$ and $R_2$:

$$\mathrm{d}C/\mathrm{d}z = \frac{2\,\pi\,\varepsilon_0}{\ln\left({R_2}/{R_1}\right)} \quad (5)$$

To obtain a compact electrode set, we've chosen electrodes of 18.00 mm and 18.50 mm of radii. With 30 mm superposition at the weighing point, the total capacitance will be 60 pF. The electrodes must be coaxial in order to get a constant capacitance gradient.

To achieve that, 2 monolithic adjustment systems with seven flexure hinges of 0.3 mm thickness can displace each outer electrode on 5 degrees of freedom (dof) (see figure 2).

Each adjustment system resolution of 1 µm and 10 µrad when using microthreaded screws; relative movements can be followed by capacitives sensors.

**Assembly**

Each capacitor is directly fixed on the balance mechanism (see figure 3), which is itself fastened on an aluminum board by a Kelvin connection and 3 springs, in order to minimize the mechanical stress on balance.

**Conclusion**

LNE has begun the conception and machining of an electrostatic force balance, aiming at realizing the mass unit at the milligram level. The balance mechanism is a monolithic simple parallelogram whereas the actuator is a cylindrical capacitor with monolithic adjustment system.

Some parts of the system are already machined and delivered, and first results should be presented at the conference.

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