

Calculation of Far Field Acoustic Radiation from Transmission Line Models of PMUT-Horn Systems

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Abstract:

In this paper, we present an extension of an acoustic transmission-line model for PMUT-horn systems that enables the prediction of acoustic pressure using a Rayleigh far-field formulation. The approach is validated against finite element method (FEM) simulations, showing consistent prediction of radiation directivity at 200 kHz. The model provides a computationally efficient alternative to FEM simulations for the design of horn-assisted PMUT systems.

Keywords: Piezoelectric-micromachined-ultrasound-transducer PMUT, acoustic horn, acoustic transmission line, rayleigh-integral, system-level modeling

Introduction and Motivation

Piezoelectric micromachined ultrasonic transducers (PMUTs) are used in airborne ultrasound applications, e.g. for the detection of persons in a room [1]. For such applications, it is necessary to improve the acoustic output and directivity. This can be achieved by coupling PMUTs to acoustic horns, which enhance impedance matching between the vibrating membrane and the surrounding air. Optimizing such systems requires efficient, yet physical models, since simulation by Finite Elements is tedious and computationally very expensive. To tackle this, previous work has demonstrated that the acoustic behavior of PMUT-horn systems can be modeled using a Butterworth-Van Dyke (BVD)-based PMUT model, combined with a transmission-line model of the horn [2]. This approach has been extended with a far-field approximation to estimate the radiated acoustic pressure along the acoustic axis [3]. However, a quantitative evaluation of the angular distribution of the radiated sound pressure and a validation against FEM simulations have not yet been implemented.

In this work, we provide a quantitative assessment of the acoustic radiation from a transmission-line-based acoustic-horn model coupled to a far-field approximation. The radiated sound field is evaluated both on and off the acoustic axis by comparison with a Rayleigh integral formulation and FEM simulations.

Modeling Approach

To compute the acoustic field, a simplified Kirchhoff integral based on the Helmholtz equation, commonly referred to as the Rayleigh integral, is employed.[4, 5]

$$p(x, y, z) = \frac{j\rho_0\omega}{2\pi} \cdot \int_S v_n \frac{e^{-j\omega A/c}}{A} dS \quad (1)$$

$$A = \sqrt{(x - x_s)^2 + (y - y_s)^2 + z^2} \quad (2)$$

Eq. (1) calculates the acoustic pressure at the observation point (x, y, z) ; (x_s, y_s, z_s) represents a point on the acoustic source. Here v_n is the normal particle velocity in the z -direction, ρ_0 the air density, c the speed of sound and A the observation distance.

Under far-field conditions (see [4, 5], i.e. for an observation distance A much larger than both, the acoustic wavelength λ and the PMUT diameter d_{PMUT} , the Rayleigh integral can be approximated by the analytical far-field expression [4, 5]:

$$p_{far}(x, y, z) = j\rho_0ckv_{throat}R_{troa}^2 \left(\frac{J_1(kR \sin\theta)}{k \sin\theta} \right) \frac{e^{(-jkA)}}{A} \quad (3)$$

J_1 is the first-order Bessel function v_{throat} is the normal velocity at the horn aperture and R_{troa} is the opening radius of the horn. Angle θ is defined as the angle between the acoustic axis (Fig. 1 (a), z -axis) and the direction towards the observation point.

Model validation is performed using finite element simulation of the system in COMSOL® Multiphysics. A uniform pressure of 100 Pa is applied for the simplified horn geometry, shown in Fig.(1) at excitation frequencies of 1 kHz and 200 kHz. The resulting pressure field is evaluated at a distance of 15 cm. At 1 kHz, the wavelength is comparable to the observation distance, which may lead to deviations due to near-field effects, whereas at higher frequencies, the far-field assumptions are better fulfilled. The input for the far-field model is obtained from the transmission-line model, while the full Rayleigh model is excited using pressure data extracted from FEM simulations.

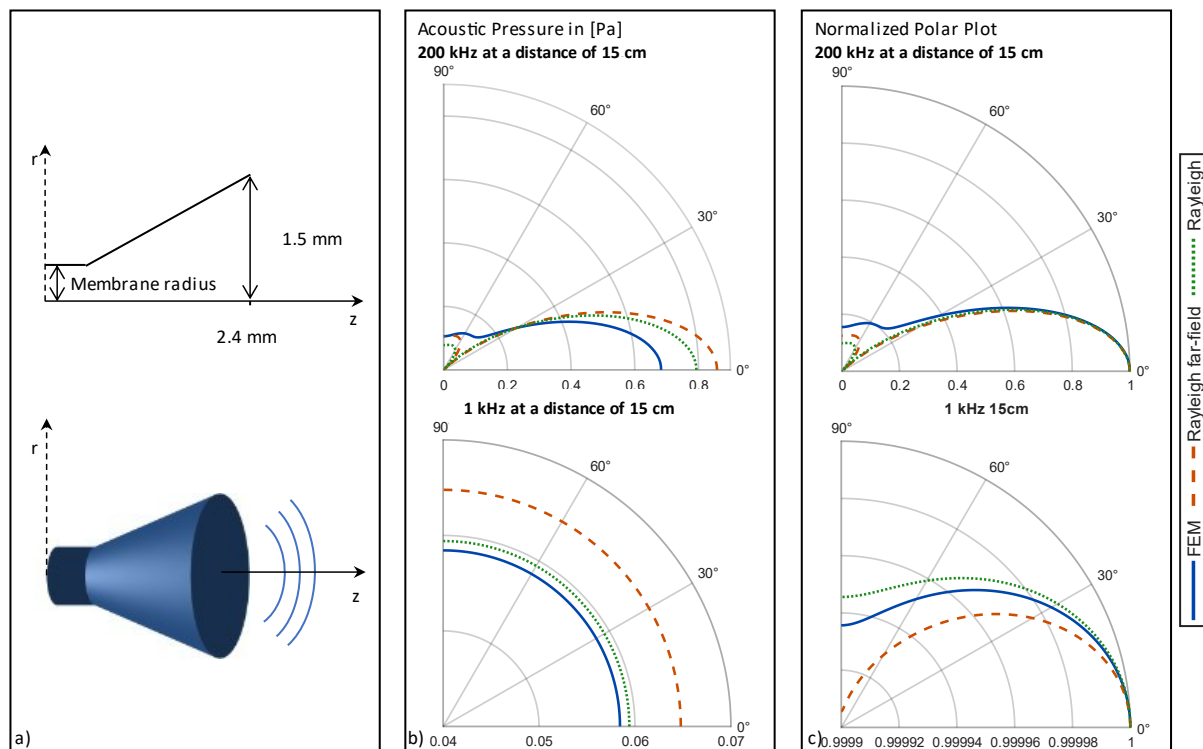


Fig.1 (a) Geometry and profile of the acoustic horn coupled to the PMUT. (b) Polar plots of the acoustic pressure calculated using FEM (blue), the Rayleigh integral (green), and the Rayleigh far-field approximation (orange) at excitation frequencies of 1 kHz and 200 kHz. (c) Corresponding normalized polar plots illustrating the radiation directivity for the different modeling approaches. The deviation at 1 kHz highlights the model's limitations, as described in the text.

Results

The comparison of the different simulation approaches demonstrates that, for acoustic pressure prediction, the transmission line model can be effectively combined with a Rayleigh far-field formulation to calculate the radiation directivity. However, the absolute acoustic pressure is overestimated by the far-field model. For an excitation frequency of 200 kHz, the Rayleigh far-field model predicts the maximum sound pressure with a worst-case deviation of approximately 25% compared to FEM simulations, while showing good agreement in radiation directivity (see Tab.1).

The discrepancies with respect to FEM results are primarily caused by modeling assumptions. In particular, radial velocity components at the horn aperture are neglected, since they are not captured by the transmission-line model. The Rayleigh integral shows improved agreement compared to the far-field approach, as it directly uses the pressure distribution obtained from FEM simulations.

Current work focuses on investigating different horn geometries and carrying out experimental measurements to characterize the samples. Further, we will extend the model to explicitly include PMUT membrane dynamics and investigate the influence of radial air-velocity components.

Tab. 1: Comparison of the different simulation methods at 200 kHz excitation frequency

Simulation-method	Directivity $D = \frac{p_{\max}}{p_{\text{avg}}}$	Difference to FEM of max. pressure in percent
FEM	2.66	-
Rayleigh	3.15	16.22
Rayleigh Far-Field	3.09	25.72

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