

Jacobian-Initialized Parametric Physics-Informed Neural Network for Anomaly Position and Size Estimation from Simulated Electrical Impedance Tomography Data

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Summary:

Real-time monitoring of the trajectory and size of the particle, droplet or bubble is relevant in many multiphase processes, but imaging techniques often fail in optically inaccessible media. In that case, electrical impedance tomography (EIT) provides a non-invasive alternative. This work presents a simulation-based study of an eight-electrode EIT sensor for monitoring a circular conductivity anomaly. We introduce a preliminary study deploying a parametric physics-informed neural network (PINN) approach that uses the center estimate from conventional Jacobian-based reconstruction to refine the trajectory and estimate the radius of an anomaly from simulated boundary-voltage measurements.

Keywords: Electrical impedance tomography, physics-informed neural network, partial differential equation, particle monitoring, inverse problems.

Introduction

In many multiphase systems, real-time monitoring of the position and size of the dispersed particle/droplet is crucial for process monitoring [1]. Experimentally, we investigated an electrical impedance tomography (EIT) cell constituted of eight electrodes (as shown in Fig. 1). Following a programmed sequence, these eight boundary electrodes excite the current in selected pairs and measure the excited responsive boundary voltages at the remaining electrode pairs to evaluate the internal conductivity distribution.



Fig. 1. Experimental eight electrode EIT cell.

However, to reconstruct the conductivity distribution information, a nonlinear and ill-posed inverse problem have to be solved. Conventional linearized methods, such as Jacobian-based reconstruction, are used to solve the EIT-governed partial differential equation (PDE). It is proven to be sufficient to reconstruct the internal conductivity field in simplified systems. However, the reconstructed anomalies are often spatially blurred due to the limited measured degrees of freedom.

This study investigates a simulation-based physics-informed neural network (PINN) approach to estimate the trajectory and radius of the moving circular conductivity anomaly. pyEIT [2] is used to generate the circular conductivity anomalies with the known position, shape and conductivity together with the corresponding boundary-voltage measurements. The initial localization of an anomaly is estimated by the Jacobian-based reconstruction, which is then refined by the subsequent parametric PINN to estimate the radius by combining the voltage consistency, governing PDE, and size-sensitive voltage-amplitude constraint.

Proposed Algorithm

A two-dimensional circular EIT phantom with eight electrodes is simulated using the pyEIT library [2]. The dataset comprises circular anomalies with different positions and radii together with their corresponding boundary-voltage measurements. For each frame, the frame voltage vector v_{frame} and the homogeneous reference vector v_{ref} are first used to obtain an initial anomaly center using a regularized Jacobian reconstruction.

In the second stage, a parametric PINN is introduced in which the conductivity distribution is represented by a circular model. This model comprises a small number of trainable geometric

parameters: the center coordinates x_c and y_c , the radius r and the anomaly conductivity σ_{in}

$$\sigma(x, y) = \sigma_{bg} + (\sigma_{in} - \sigma_{bg})S(r - d(x, y)), \quad (1)$$

where

$$d(x, y) = \sqrt{(x - x_c)^2 + (y - y_c)^2}, \quad (2)$$

and S denotes a smooth circular boundary. The Jacobian estimate initializes the center coordinates, such that $x_c = x_{jac}$ and $y_c = y_{jac}$, while the same initial radius is used for all frames.

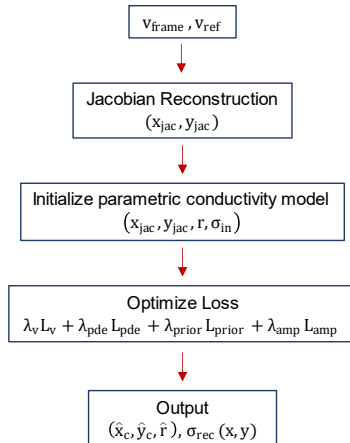


Fig. 2. Workflow of the proposed Jacobian-initialized parametric PINN.

As shown in Fig. 2, the potential-network weights and anomaly parameters are then optimized jointly. The loss terms L_v and L_{pde} ensure agreement with the simulated data and physical consistency, whereas L_{prior} preserves the Jacobian-based initialization. The amplitude loss L_{amp} provides sensitivity to the anomaly size. The output layer returns the refined center coordinates, estimated radius, and corresponding conductivity distribution.

Results

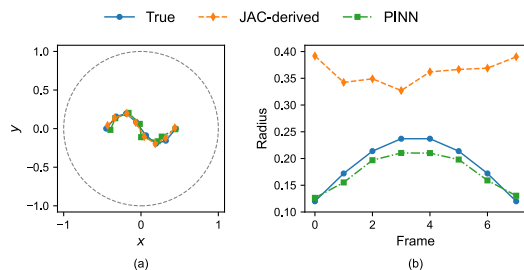


Fig. 3. Comparison over eight simulated frames. True, Jacobian-derived and PINN-estimated (a) center trajectories, and (b) size of anomalies.

The comparison of recovered anomaly trajectory and anomaly size is illustrated in Fig. 3. All three trajectory curves indicate that the center

estimation is almost close to the true center location (Fig. 3(a)). Fig. 3(b) shows that the effective radius estimation of the proposed parametric PINN remains closer to the ground truth in comparison to the conventional Jacobian-based reconstruction. Fig. 4 demonstrates that the PINN recovers the approximate extent of an anomaly. The absolute error maps suggest that the largest deviations appear near the circular interface.

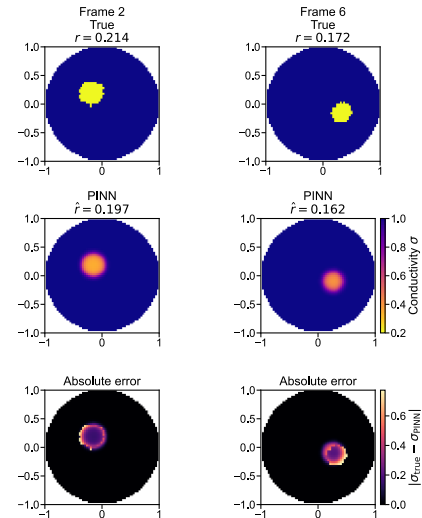


Fig. 4. Representative parametric PINN reconstructions for frame 2 and frame 6. Each row shows the ground truth, PINN estimation, and an absolute reconstruction error.

Conclusion

The Jacobian-based reconstruction localized the anomaly efficiently, but it blurred the anomaly interface and overestimated the effective radius. In comparison, the parametric PINN preserved the estimated trajectory and recovered the radius more closely to the ground truth. The results demonstrate the robustness of combined Jacobian initialization and physics-informed parameter estimation for position and size monitoring.

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References

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