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Sensitivity Analysis and Material Parameter Estimation of a Pre-Stressed Langevin Transducer

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Abstract: In this study, the stress-dependency of piezoelectric material parameters is investigated. A finite element model is developed to analyse the electrical impedance response of a mechanically pre-stressed Langevin transducer. The experimental data indicate a single dominant resonance, which limits the estimation of piezoceramic material parameters under different mechanical stresses. In addressing this issue, a sensitivity analysis is employed to identify the parameters with the greatest influence on the resonance. Based on this, a subset of the stress-dependent material parameters of the piezoelectric components can be identified in an inverse procedure.

Keywords: Piezoceramic rings, Langevin transducer, Sensitivity analysis, Parameter estimation

Motivation

Piezoelectric ceramics find application in high-power ultrasonic sensors and actuators, where they are subjected to significant electrical and mechanical stresses, leading to non-linear effects. An approach to analyse such non-linear behaviour involves shifting the operating points by varying the static mechanical load and characterising the linear behaviour in each operating point. This method allows for an approximation of the material's non-linear characteristics. Nevertheless, the realisation of a homogeneous mechanical load is still a considerable challenge [1, 2]. In Langevin transducers, which are typically employed in high-power applications, the piezoceramics are pre-stressed axially as homogeneously as possible in order to avoid mechanical tensile stresses. However, this approach increases the complexity of the simulation model, as it necessitates the consideration of additional metallic components and a second ceramic element.

The experimental measurements, and consequently the simulation results (cf. Fig. 4), demonstrate that only a single resonance is clearly discernible in the electrical impedance spectrum within the considered frequency range. This is due to the fact that radial and torsional modes are barely pronounced in this configuration. The resonance is a so-called thickness-extensional mode, which is located in a frequency range in which transducers of this kind are typically operating [3]. This limitation makes it difficult to estimate a complete set of piezoelectric material parameters in an inverse procedure, which requires additional resonances [4].

A two-dimensional simulation model is developed for the purpose of simulating the transducer's frequency-dependent electrical impedance. A detailed analysis of the resulting impedance data is performed, paying particular attention to the resonance behaviour, which is evidently different for a transducer constructed with multiple components than for a single, unconstrained piezoceramic ring. The material parameters with the greatest influence on the resonance are identified by means of a sensitivity analysis of the simulated impedance data. This, in turn, allows for a targeted parameter estimation process based on measurements under varying load conditions.

Numerical model

Mathematically, the behaviour of piezoelectric materials can be modelled as functions of electrical and mechanical quantities

$$\boldsymbol{T} = -\underline{\boldsymbol{e}}^{\mathrm{t}}\boldsymbol{E} + \underline{\boldsymbol{c}}^{\mathrm{E}}\boldsymbol{S} \tag{1}$$

$$\boldsymbol{D} = \underline{\boldsymbol{\varepsilon}}^{\mathrm{S}}\boldsymbol{E} + \underline{\boldsymbol{e}}\boldsymbol{S}, \tag{2}$$

where $\boldsymbol{T}$ represent the mechanical stress, $\boldsymbol{S}$ the mechanical strain, $\boldsymbol{E}$ the electrical field strength, and $\boldsymbol{D}$ the electrical displacement. The mechanical stiffness matrix $\underline{\boldsymbol{c}}^{\mathrm{E}}$, the permittivity matrix $\underline{\boldsymbol{\varepsilon}}^{\mathrm{S}}$, and the piezoelectric coupling matrix $\underline{\boldsymbol{e}}$ are the matrices of the material parameters of interest. Employing the Voigt notation [5] and taking into account the symmetry conditions of a transversely isotropic piezoceramic material, the material parameter matrices are given as follows:

$$\underline{\boldsymbol{c}}^{\mathrm{E}} = \begin{bmatrix} c_{11}^{\mathrm{E}} & c_{12}^{\mathrm{E}} & c_{13}^{\mathrm{E}} & 0 & 0 & 0 \\ c_{12}^{\mathrm{E}} & c_{11}^{\mathrm{E}} & c_{13}^{\mathrm{E}} & 0 & 0 & 0 \\ c_{13}^{\mathrm{E}} & c_{13}^{\mathrm{E}} & c_{33}^{\mathrm{E}} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44}^{\mathrm{E}} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44}^{\mathrm{E}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{(c_{11}^{\mathrm{E}} - c_{12}^{\mathrm{E}})}{2} \end{bmatrix} \tag{3}$$

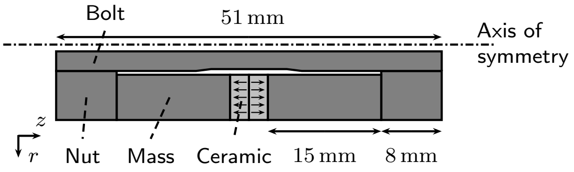

Fig. 1: Two-dimensional, rotationally symmetric simulation model for a Langevin transducer, composed of two oppositely polarised piezoceramic rings (light grey), steel masses, nuts and a hollow bolt.

$$\underline{\boldsymbol{e}} = \begin{bmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{15} & 0 & 0 \\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0 \end{bmatrix} \tag{4}$$

$$\underline{\boldsymbol{\varepsilon}}^{\mathrm{S}} = \begin{bmatrix} \varepsilon_{11}^{\mathrm{S}} & 0 & 0 \\ 0 & \varepsilon_{11}^{\mathrm{S}} & 0 \\ 0 & 0 & \varepsilon_{33}^{\mathrm{S}} \end{bmatrix} . \tag{5}$$

Including damping, e.g. Rayleigh damping, characterised by the parameters $\alpha_{\mathrm{M,p}}$ and $\alpha_{\mathrm{K,p}}$ [6], and the density ρ_{p} of the piezoelectric material, a total of 13 relevant piezoceramic parameters are given:

$$\boldsymbol{p}_{\mathrm{mat,piezo}} = \big[\rho_{\mathrm{p}}, c_{11}^{\mathrm{E}}, c_{12}^{\mathrm{E}}, c_{13}^{\mathrm{E}}, c_{33}^{\mathrm{E}}, c_{44}^{\mathrm{E}}, \varepsilon_{11}^{\mathrm{S}}, \varepsilon_{33}^{\mathrm{S}}, e_{15}, e_{31}, e_{33}, \alpha_{\mathrm{M,p}}, \alpha_{\mathrm{K,p}}\big] . \tag{6}$$

The calculation of frequency-dependent impedance for a given geometry and set of material parameters necessitates the employment of a simulation model. The results can be achieved by means of a finite element approximation [7], for instance, within the simulation tool openCFS (Coupled Field Simulation) [8]. The simulation model for the Langevin transducer implemented in this study is illustrated in Fig. 1. It includes a pair of piezoceramic rings, which are compressed by hollow, cylindrical steel masses, nuts and a hollow bolt made of steel. The thickness of the piezoceramic rings (PIC184, *PI Ceramic*, Germany) is $2.5\,\mathrm{mm}$, whilst the outer radius is $10\,\mathrm{mm}$ and the inner radius $4\,\mathrm{mm}$. Further dimensional information can be found in the figure. The electrical contact is facilitated by $0.1\,\mathrm{mm}$ copper plates, which are omitted in this model. The parameters in Eq. (6) are used for the description of the piezoceramic rings. The parameters for the steel components are as follows:

$$\boldsymbol{p}_{\mathrm{mat,steel}} = \big[\rho_{\mathrm{s}}, c_{\mathrm{L,s}}, c_{\mathrm{T,s}}, \alpha_{\mathrm{M,s}}, \alpha_{\mathrm{K,s}}\big] . \tag{7}$$

The density is denoted by ρ_{s}, the longitudinal velocity by $c_{\mathrm{L,s}}$, the transversal velocity by $c_{\mathrm{T,s}}$, and the damping parameters by $\alpha_{\mathrm{M,s}}$ and $\alpha_{\mathrm{K,s}}$, analogous to $\alpha_{\mathrm{M,p}}$ and $\alpha_{\mathrm{K,p}}$. An alternative description could be provided using the Young's modulus E, and the Poisson ratio ν of the steel components.

Tab. 1: Initial values for PIC184 piezoceramic material and for steel parameters.

Parameter	Value	Parameter	Value
c_{11}^{E}	$140.22\,\mathrm{GPa}$	$\varepsilon_{11}^{\mathrm{S}}$	$5.65\,\mathrm{nF\,m^{-1}}$
c_{12}^{E}	$76.19\,\mathrm{GPa}$	$\varepsilon_{33}^{\mathrm{S}}$	$5.58\,\mathrm{nF\,m^{-1}}$
c_{13}^{E}	$80.2\,\mathrm{GPa}$	$\alpha_{\mathrm{M,p}}$	$6.49 \cdot 10^{3}\,\mathrm{s^{-1}}$
c_{33}^{E}	$126.24\,\mathrm{GPa}$	$\alpha_{\mathrm{K,p}}$	$0.24\,\mathrm{ns}$
c_{44}^{E}	$25.41\,\mathrm{GPa}$	$c_{\mathrm{L,s}}$	$6130.86\,\mathrm{m\,s^{-1}}$
e_{15}	$10.65\,\mathrm{C\,m^{-1}}$	$c_{\mathrm{T,s}}$	$3277.08\,\mathrm{m\,s^{-1}}$
e_{31}	$-5.39\,\mathrm{C\,m^{-1}}$	$\alpha_{\mathrm{M,s}}$	$7.41 \cdot 10^{-8}\,\mathrm{s^{-1}}$
e_{33}	$14.2\,\mathrm{C\,m^{-1}}$	$\alpha_{\mathrm{K,s}}$	$0.1\,\mathrm{ns}$
ρ_{p}	$7750\,\mathrm{kg\,m^{-3}}$	ρ_{s}	$7700\,\mathrm{kg\,m^{-3}}$

Due to the transducer's design, two oppositely polarised piezoceramics are utilised. It is therefore essential that the polarisation direction is reflected in the material parameters. A straightforward calculation demonstrates that this can be denoted by a negative sign in the matrix of piezoelectric constants Eq. (4). The two-dimensional model is rotationally symmetric, as indicated by the dashed-dotted line situated along the longitudinal z-axis, which serves to reduce the computational time. An investigation of the vibration modes of this model reveals a certain degree of analogy to results from simpler models in previous studies [3], which indicates the reliability of the model used here.

Sensitivity analysis

The objective of a sensitivity analysis is to identify the material parameters that have a significant impact on the numerically calculated impedance. For these parameters, the probability of good estimation is higher using an inverse procedure, although the frequency range is limited. The numerical model of the transducer is evaluated using the finite element method [8] to simulate the complex impedance within the specified frequency range. The full set of material parameters is used, with each parameter being modified independently. The piezoceramic parameters for the simulation are obtained from the estimation of an unconstrained sample [4], and the parameters for steel are taken from data sheets [9]. The used parameters are given in Tab. 1. The deviation between two different simulated spectra is defined as a measure of sensitivity as follows:

$$\varUpsilon = \left[\sum_{i} \big||Z_{\mathrm{ref},i}| - |Z_{\mathrm{var},i}|\big|^{2}\right]^{\frac{1}{2}} . \tag{8}$$

In this context Z_{ref} is the reference impedance vector that serves as a baseline against which other, varying

Tab. 2: Resulting sensitivities of piezoceramic material and steel parameters.

Parameter	Sensitivity Υ / Ω	Parameter	Sensitivity Υ / Ω
c_{11}^{E}	9210	$\varepsilon_{11}^{\mathrm{S}}$	13
c_{12}^{E}	4920	$\varepsilon_{33}^{\mathrm{E}}$	20 079
c_{13}^{E}	39 865	$\alpha_{\mathrm{M,p}}$	59
c_{33}^{E}	44 893	$\alpha_{\mathrm{K,p}}$	17
c_{44}^{E}	347	$c_{\mathrm{L,s}}$	46 591
e_{15}	91	$c_{\mathrm{T,s}}$	124 320
e_{31}	6009	$\alpha_{\mathrm{M,s}}$	4706
e_{33}	31 284	$\alpha_{\mathrm{K,s}}$	$5 \cdot 10^{-8}$

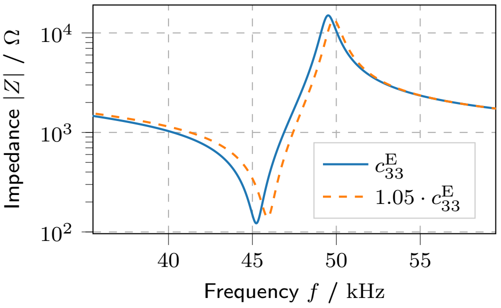

Fig. 2: Influence of an increase by $5\,\%$ of the c_{33}^{E} parameter on the electrical impedance of the simulated Langevin transducer.

impedances are compared. The impedance vector Z_{var} is any impedance vector with a material parameter that varies by $5\,\%$. The index i is used to correspond to the sampled frequency points of the impedance vectors.

The resulting deviations are shown in Tab. 2 for each parameter. The absolute sensitivity offers limited interpretative value due to its dependence on the number of sampled frequency points. However, it can be interpreted relatively to other values. These deviations are attributed to shifts in the impedance, as illustrated exemplary for c_{33}^{E} in Fig. 2. It can be observed that the piezoceramic parameters in the longitudinal 33-direction, i.e. in the direction of the z-axis in Fig. 1, are the parameters with the highest sensitivity measure. This phenomenon also aligns with the fact that the exciting electrical field is primarily present in this direction. This observation indicates that, within the constrained frequency range, estimation of the piezoceramic parameters in 33-direction is a feasible option.

Tab. 3: Estimated steel material parameters.

Parameter	Value	Parameter	Value
$c_{\mathrm{L,s}}$	$6123.79\,\mathrm{m\,s^{-1}}$	$\alpha_{\mathrm{M,s}}$	$3.99 \cdot 10^{-8}\,\mathrm{s^{-1}}$
$c_{\mathrm{T,s}}$	$3473.65\,\mathrm{m\,s^{-1}}$	$\alpha_{\mathrm{K,s}}$	$5.46\,\mathrm{ps}$

Tab. 4: Estimated piezoceramic material parameters in 33-direction for PIC184.

Parameter	Mechanical Stress		
	$40\,\mathrm{MPa}$	$50\,\mathrm{MPa}$	$60\,\mathrm{MPa}$
c_{33}^{E} / GPa	126.29	128.37	131.73
e_{33} / $\mathrm{C\,m^{-1}}$	16.23	17.77	18.31
$\varepsilon_{33}^{\mathrm{E}}$ / $\mathrm{nF\,m^{-1}}$	6.9	7.18	7.3

Material parameter estimation

The simulated impedance, using the initial values from Tab. 1, shows a significant deviation from the measurements (cf. Fig. 3). A possible explanation for this is the fact that the minimal mechanical load of $30\,\mathrm{MPa}$ is necessary for the parts of the transducer to be physically in a full contact [10]. As a consequence, the estimation of the steel parameters is performed first in an inverse procedure, with the parameters given priority according to their impact on the impedance. The sequence of parameters to be estimated is therefore firstly $c_{\mathrm{T,s}}$, subsequently $c_{\mathrm{L,s}}$, and finally the damping parameters (ref. Tab. 2). The resulting steel parameters from Tab. 3 are then used for the subsequent optimisation of the piezoceramic material parameters in 33-direction simultaneously, using measurements for different mechanical loads. The resulting estimated values for piezoceramic materials are presented in Tab. 4. It is evident that the values of the estimated material parameters successively increase with mechanical load. The application of the estimated parameters in simulating an impedance and subsequent comparison with measurement data reveals a high degree of agreement, as can be seen in Fig. 3.

When simulating a wider frequency range, the problem of prominent resonances can be observed once more in Fig. 4. It is evident that, particularly in the case of simulated impedance, the first resonance can be adequately represented, while subsequent resonances are more challenging to assign and to interpret. These are assumed to be higher harmonics. Furthermore, there are modes which can not be captured by the 2D simulation model, including (like) bending and circumferential modes [11].

Conclusion

In this study, stress-dependent piezoelectric material parameters are identified. A simulation model of a Langevin transducer is created, which enables the nu-

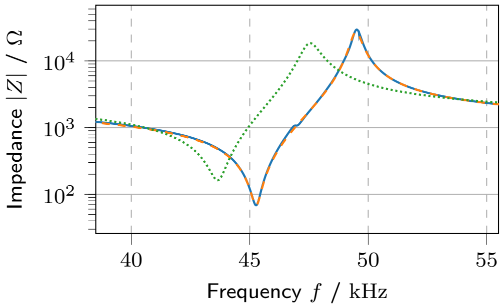

Fig. 3: Comparison of measured (——) and simulated impedance data of a Langevin transducer using the estimated (- - -) and initial (·········) material parameters for $40\,\mathrm{MPa}$.

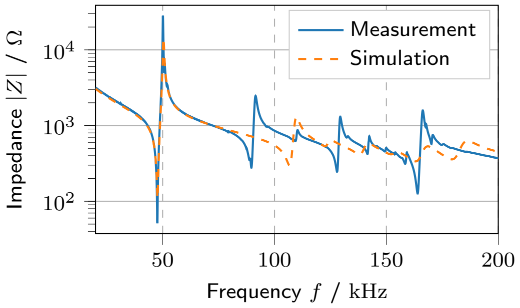

Fig. 4: Comparison of measured and simulated impedance data of a Langevin transducer for a mechanical stress of $60\,\mathrm{MPa}$ *using the estimated material parameters in a wider frequency range.*

merical calculation of electrical, frequency-dependent impedances. The use of numerical simulation is advantageous in this context, as it facilitates a more profound comprehension of the processes within the given structure. Consequently, it is feasible to investigate vibration modes in greater detail and to perform sensitivity studies to identify the most advantageous material parameters for estimation. The findings contribute to a more refined understanding of the role of mechanical pre-stress in piezoelectric transducers and provide a systematic approach for a parameter identification under constrained measurement conditions. Despite the restriction to a single resonance, it is possible to identify a subset of the piezoceramic material parameters. The results presented herein demonstrate a change in these material parameters with increasing mechanical load. It provides an impression of the non-linearities present in this operating case and can subsequently be used to model these non-linearities. Further research could investigate the potential for enhanced interpretation and utilisation of the less pronounced resonances to estimate additional material parameters.

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References

[1] C. Scheidemann et al. "Influence of Temperature and Pre-Stress on the Piezoelectric Material Behavior of Ring-Shaped Ceramics". In: *7th International Conference on Advanced Electromaterials (ICAE 2023) and International Conference on Functional Materials & Devices 2023 (ICFMD 2023)*. 2023.

[2] O. Friesen et al. "Untersuchung piezoelektrischer Materialeigenschaften unter hydrostatischer Last". In: *Fortschritte der Akustik - DAGA 2024*. Ed. by Deutsche Gesellschaft für Akustik e.V. 2024, pp. 1117–1120.

[3] A. Iula et al. "Finite element three-dimensional analysis of the vibrational behaviour of the Langevin-type transducer". In: *Ultrasonics* 40.1-8 (2002), pp. 513–517.

[4] O. Friesen et al. "Estimation of piezoelectric material parameters of ring-shaped specimens". In: *tm - Technisches Messen* (2024). DOI: 10.1515/teme-2024-0107.

[5] W. Voigt. *Lehrbuch der Kristallphysik (mit Ausschluss der Kristalloptik)*. 1966th ed. Vieweg+Teubner Verlag, Jan. 1928. ISBN: 978-3-663-15316-0.

[6] N. Feldmann et al. "Modelling damping in piezoceramics: A comparative study". In: *tm - Technisches Messen* 88.5 (2021), pp. 294–302. DOI: 10.1515/teme-2020-0096.

[7] M. Kaltenbacher. *Numerical simulation of mechatronic sensors and actuators*. Vol. 2. Springer, 2007.

[8] S. Schoder and K. Roppert. *openCFS: Open Source Finite Element Software for Coupled Field Simulation – Part Acoustics*. 2022. DOI: 10.48550/arXiv.2207.04443. arXiv: 2207.04443 `[math.NA]`.

[9] G. Lucefin. "Lucefin steel". In: *Technical card, revision* (2013).

[10] F. Arnold and S. Mühlen. "The influence of the thickness of non-piezoelectric pieces on prestressed piezotransducers". In: *Ultrasonics* 41.3 (2003), pp. 191–196.

[11] B. S. Aronov. "Piezoelectric circular ring flexural transducers". In: *The Journal of the Acoustical Society of America* 134.2 (Aug. 2013), pp. 1021–1030. DOI: 10.1121/1.4812760.