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Phononic Crystallography of Fibre-Reinforced Polymers using Broadband Acoustic Waves

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Abstract: Assuming that fibre-reinforced polymers can be modelled as two-dimensional phononic crystals, the present study proposes an approach to experimentally determine their crystallographic properties. The results closely resemble classical crystallographic methods, such as X-ray diffraction, but allow for the characterisation of macroscopic structures by non-destructive, acoustic means. The proposed method is based on the evaluation of spatio-temporal measurement data of broadband guided acoustic waves in the frequency and wavenumber domain.

Keywords: Waveguides, Periodic structure, Phononic crystals, Crystallography, Fibre-reinforced polymers

Motivation

Fibre-reinforced polymers have attracted considerable interest due to their exceptional mechanical properties, including high strength-to-weight ratios, which renders them ideal for a broad spectrum of engineering applications. The material composition and orientation of the reinforcing fibres permit a flexible and application-specific design of the composite material for the respective use case. In addition, composites are distinguished by their high resistance to corrosion and fatigue. Fibre-reinforced polymers exhibit an orthogonal woven structure and are distinguished by variations in the number of crossed fibres or the fibre distribution [1]. Typically, fibre-reinforced polymers are fabricated as laminated composites, with multiple layers. However, during subsequent processing and forming operations, local deviations in periodicity and alterations in the geometry of the fibre weave can occur, potentially impacting the mechanical properties.

The present study employs pulsed laser radiation as a non-destructive technique to induce broadband acoustic waves within fibre-reinforced polymer samples. The samples are conceptualised as phononic crystals, wherein their internal structure and inherent periodicity influence their acoustic behaviour [2]. Whilst phononic crystals are typically deliberately designed to exhibit engineered band gaps for specific functional purposes, such as vibration isolation or wave guiding, this work is focused on analysis of existing structures. The proposed method of evaluating guided acoustic wave measurement data from these samples enables the determination of the phononic crystals properties, such as the unit cell geometry, by evaluating the material's periodicity, thereby providing a framework for the quantification of structural properties.

Experimental Procedure

The measurement setup shown in Fig. 1 is utilised for the analysis of the properties of guided acoustic waves in fibre-reinforced polymers. Pulsed ($1.5\,\mathrm{ns}$), infrared laser radiation is directed and focused onto a line on the sample by a cylindrical lens. The energy density is sufficiently low to prevent damage to the sample, resulting only in a fast, localised increase in temperature. The thermoelastic effect results in the excitation of broadband acoustic waves, which propagate through the sample. The excited acoustic waves are detected by a piezoelectric transducer, which utilises a strip-shaped piezoelectric ceramic as the active element. In order to analyse wave propagation in the spatial domain, the excitation location in x-direction is variable. A recorded dataset is two-dimensional, resolved in both space and time.

The transformation of the data into the wavenumber-frequency domain can be achieved applying a two-dimensional Fourier transform. [3] The result can then be interpreted similarly to a dispersion diagram, with ridges indicating propagating modes. In the dispersion diagrams of fibre-reinforced polymers, the propagating modes exhibit repetitions in the wavenumber range with a period of Δ_{k_x} (Fig. 2). This effect directly results from the periodic internal structure of the samples [5], [6]. The spatial period a of the sample can be calculated using the following reciprocal relationship [7]

$$a = \frac{2\pi}{\Delta_{k_x}}. \tag{1}$$

To evaluate the measurement data with respect to their periodicity, the autocorrelation of the dispersion diagrams is calculated in the direction of the wavenumber and averaged over the frequency (Fig. 3). Each

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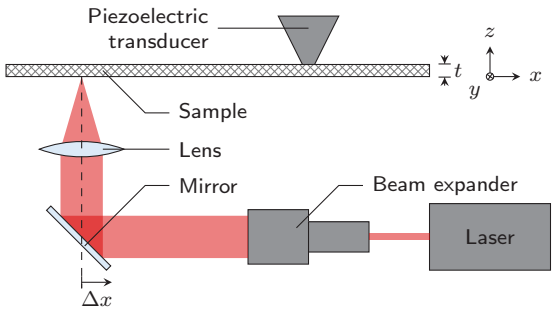

Fig. 1: Experimental setup for the excitation and detection of broadband acoustic waves in plate-like samples with variable excitation position [4].

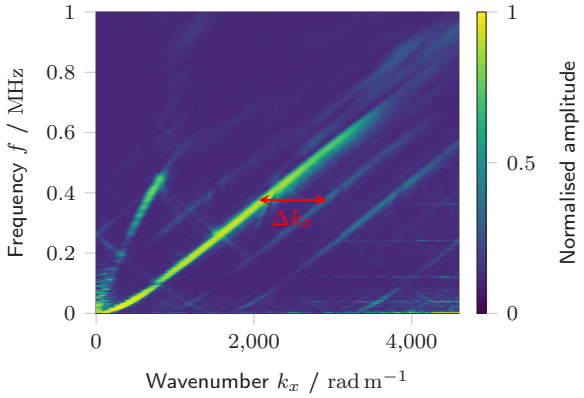

Fig. 2: Measured dispersion diagram of a fibre-reinforced polymer sheet with plain weave. The sample is three-layered and has a thickness of 1.5 mm.

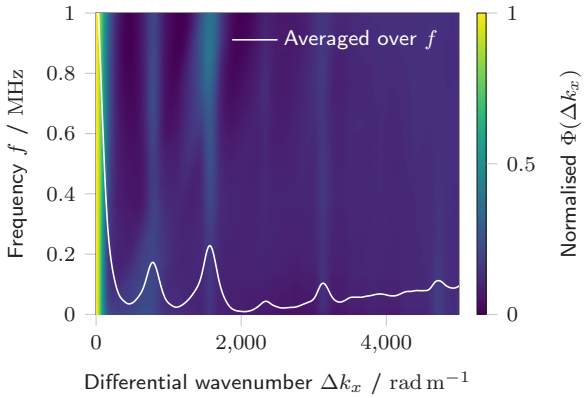

Fig. 3: Autocorrelation along the wavenumber direction of the dispersion diagram from a fibre-reinforced polymer with plain weave (Fig. 2), and averaged over the frequency. [8]

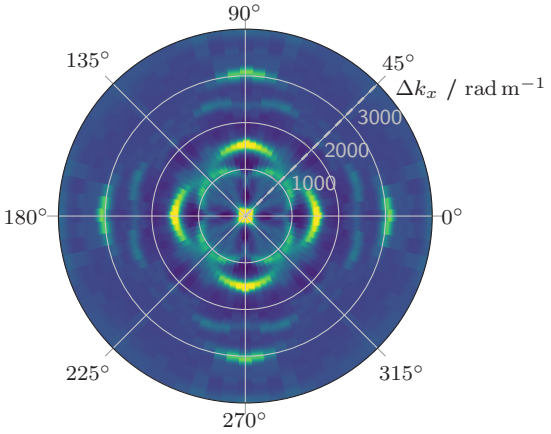

Fig. 4: Frequency-averaged autocorrelation functions as a function of the angel α from a sample with plain weave.

maximum thus represents a period of the measurement data in the wavenumber regime. The distances $\Delta_{k_x,i}$ of the maxima can be averaged and converted into the spatial period a using Eq. (1). Consequently, a robust methodology exists for determining the size of the unit cell [8].

Unit cell geometry

In order to ascertain the geometry of the unit cell, the proposed evaluation of the periodicity in wavenumber regime is conducted as a function of an angle α. α is defined as the angle between the measurement direction (x-direction) and the direction of one of the fibre bundles in the wave. The frequency-averaged autocorrelation functions (compare Fig. 3) as a function of the angle α for a sample with plain wave is shown in Fig. 4. Remapping the radial axis using the reciprocal relationship from Eq. (1) results in a depiction in the spatial regime (Fig. 5). The intensities of these peaks are indicative of the geometry of the present unit cell of a plain weave. The unit cell under consideration is square, with a side measuring $a = 8.1\,\mathrm{mm}$, which aligns well with optical measurements of the sample. Fig. 5 also shows a larger square that is rotated by $45°$. The current assumption as to why the square is visible is that these depictions visualise reflection planes in the samples, which also occur at an angle of $45°$.

The reciprocal relationship $a = \frac{2\pi}{\Delta k_x}$ to map the wavenumber data to the spatial domain is used instead of a Fourier transformation as an approximation because manual handling of the sample in the measurement setup currently does not yield reliable phase information. It is evident that this mapping does not result in the restoration of the original spatial signal; rather, it provides an intuitive spatial structure. A

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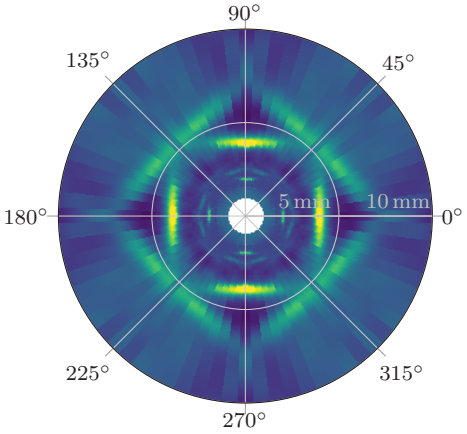

Fig. 5: Result of the proposed method to characterise the internal periodic structure of a sample with plain weave showing a symmetrical, square unit cell geometry.

consequence of the inverted mapping under consideration is that repetitions or artefacts at smaller values of a appear which are not interpretable.

The results of the proposed method for the analysis of the internal, periodic structure bear a strong resemblance with images from X-ray crystallography, which is a method of visualising the atomic arrangements of crystals by detecting diffraction patterns [9]. It is thus referred to as phononic crystallography, because it allows for the analysis of phononic crystals and their dispersive properties, like reflection planes and the unit cell geometry.

Results

To assess the capability of the proposed method to determine the structural properties of a given sample with internal periodic structure, it applied to detect fibre weave deformation. The samples consist of a single-layer of glass fibre 2/2 twill weave embedded in a polypropylene matrix. The measurements are conducted in a range of $0°..180°$ and then repeated due to the symmetry of the weave. As illustrated in Fig. 6, the applying the proposed method shows the quadratic unit cell of the twill wave, with a side length of $a = 9.58\,\mathrm{mm}$. This corresponds to the expected value, which can be determined visually.

In a 2/2 twill weave, fibres bundles in one direction always cross over two fibre bundles in the other direction. It is assumed, that reflection of acoustic waves occur when the fibres transition between layers. The dispersion diagram of the sample with plain weave shows repetitions at a and $\frac{a}{2}$ are discernible (respectively Δk_x and $2\Delta k_x$). The repetitions at a are more prominent in the measurement data and are thus evaluated. Furthermore, the ridges at $135°$

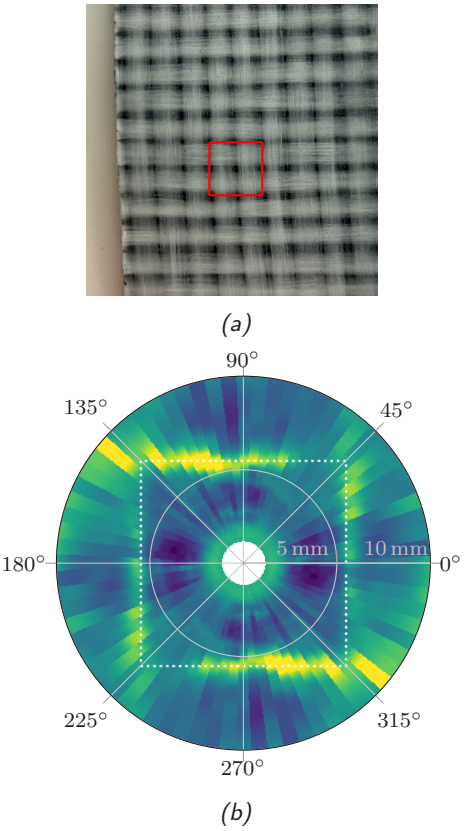

Fig. 6: Result of the proposed method to characterise the internal periodic structure of a sample with twill weave (a) showing a square unit cell geometry with asymmetric intensity (b).

are more pronounced than at $45°$; this phenomenon corresponds with the asymmetry of the weave with respect to rotations of $90°$. A comparison with the results from the plain weave (Fig. 5) shows that the proposed method can thus also infer basic properties of the pattern of the weave. The exact correspondence of weave pattern and the results are subject of future studies.

The same analysis is conducted on a distorted single-layer sample with twill weave (Fig. 7). The composition of this sample is identical to that of the reference sample. It is evident that the distortion of the weave in a single direction results in the unit cell assuming a parallelogram shape. It is imperative to demonstrate that the measurement results of the proposed method show this geometric change. The result (Fig. 7b) shows that the distortion of the unit cell is clearly detected and determined to be approximately $20°$.

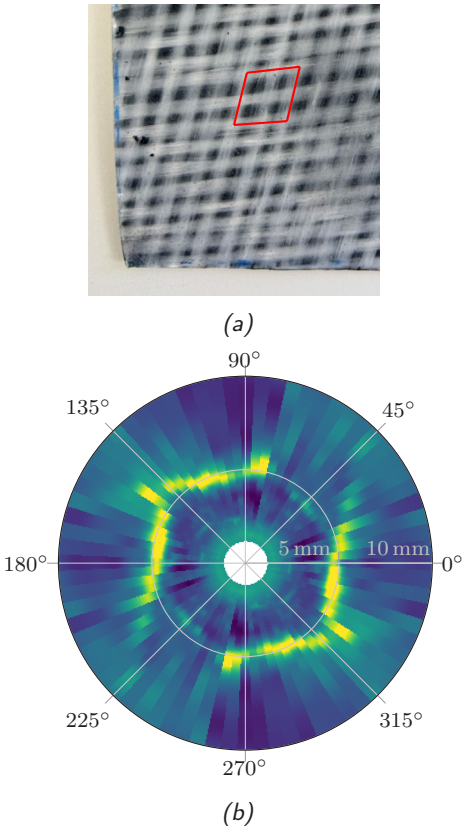

Fig. 7: Result of the proposed method to characterise the internal periodic structure of a sample with distorted twill weave (a) showing a unit cell with parallelogram shape with asymmetric intensity (b).

Conclusion

Using the proposed method, the geometry and size of unit cells of different weave patterns in fibre-reinforced polymers can be determined by evaluating the dispersive behaviour of acoustic plate waves. This is especially evident when evaluating samples with non-rectangular unit cells. Indications for the asymmetry of specific weave pattern can also be found in the results.

Subsequent research will investigate local distortion or changes in periodicity. Synthetically created periodic structures will be analysed to further validate the findings of this study. This facilitates the investigation of the influence of a small aperiodicity on phononic crystallography. A potential extension of the method is to quantitatively analyse the maxima in the autocorrelation of the dispersion data.

A full empirical band structure of the sample can also be inferred from the measurement data, which will be evaluated for material modelling purposes in future studies.

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