DOI 10.5162/Ultrasonic2025/A1-b1

Guided Acoustic Waves on Periodic Structured Plates: from Theory to Application

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Abstract: Guided acoustic waves are well studied for unstructured plates. In the last years the view of researchers has been extended to phononic crystals consisting of periodic structured or combined materials in the direction of sound propagation. In this work, the influence of one-sided and two-sided structured geometries on Lamb waves is investigated by simulations and laser Doppler vibrometry. In addition to the analysis of band structures, the transmission, reflection and mode conversion behavior of Lamb waves is characterized for finite structures.

Keywords: Guided Acoustic Waves, Lamb Waves, Phononic Crystals, Finite Elements Method, Laser Doppler Vibrometry

Introduction

Guided acoustic waves (GAW), including Lamb, Rayleigh, and quasi-Scholte waves, are used in various sensor and actuator applications, such as deposit detection, flow rate measurement, and non-destructive testing or fluid manipulation [1]. Recent acoustic research focuses on phononic crystals (PCs) for the manipulation of acoustic waves [2]. These investigations are facilitated by advanced simulation methods. In particular, the finite element method (FEM) enables a fast characterization of various geometries. This development makes studies more efficient, reducing the need for extensive physical experiments. Nevertheless, this option has been observed to demonstrate the tendency of structures to be examined exclusively through simulation, yielding geometries which are difficult or impossible to manufacture [3]. In this work, the interaction of Lamb waves with structured metal plates is investigated. For the first analysis, the band structure diagrams for different symmetric and antisymmetric structures are calculated and characterized. For experimental validation, a laser Doppler vibrometer (LDV) is used. Additionally, the interaction of Lamb waves with a finite number of structure elements is investigated by frequency domain simulations. Here, the reflection, transmission, and mode conversion behavior of Lamb waves at such structures is calculated. Finally, challenging aspects of experimental validation as well as the possibilities of using such structures in combination with fluid exposure are discussed.

Methods

In this paper, a model based on a unit cell (UC) was used for the calculation of band structures [4]. In the case of an unstructured plate, the two-dimensional finite element model for the unit cell consists exclusively of a rectangle with height H_{uc} and width B_{uc}. Floquet boundary conditions can be utilized to calculate the cell dispersion behavior for predefined wave numbers k_x by means of eigenfrequency analysis. According to Eq. (1) the dimensionless parameter k has to be swept from 0 to 1 to evaluate all relevant values of k_x in the study.

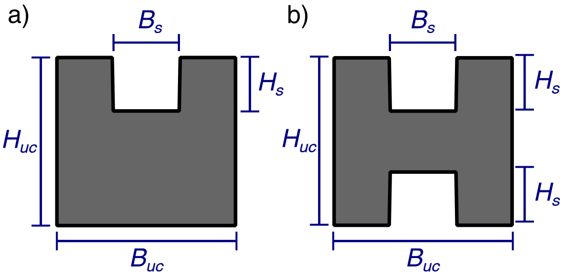

Fig. 1: Sketches of the unit cells used for the band structure analysis with a) single-sided and b) double-sided structures.

$$k_x = \frac{\pi}{B_{\mathrm{uc}}} k \quad \text{with} \quad k \in [0,1] \tag{1}$$

Lamb waves, which ordinarily propagate on flat plates, are characterized according to their order of oscillation and the symmetry behavior in the plate. A standard distinction is made between symmetric and antisymmetric modes [5]. In order to investigate the influence of the symmetry behavior of the periodic structuring on the different Lamb waves, two types of UCs were investigated in this work. These grooves were denoted either single-sided or double-sided, depending on their orientation relative to the sample's

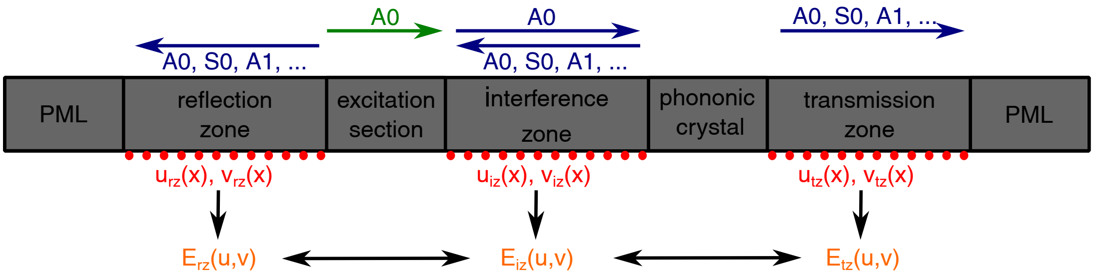

Fig. 2: Sketch of the simulation for the evaluation of the transmission and reflection behavior of limited structures.

surface. The parameters B_s (width) and H_s (height) were introduced to describe the structured geometries (Fig. 1). In the simulations, the dimensions of the geometry were gradually increased starting from the simple unstructured plate in order to be able to analyze the progressive changes in the derived band structure diagrams. The geometric parameters used for the UC are listed in Tab. 1.

The former considered band structure is theoretically only valid for infinitely long periodic PCs. To study the performance of limited structures, a new frequency domain model was built. Therefore, a long plate geometry was generated consisting of different zones (see Fig. 2). In these simulations, a right-wards traveling A0 mode was generated in the excitation zone. By passing the interference zone the limited PC was reached causing reflections, transmission, and mode conversion. Perfectly matched layers (PML) at both ends of the panel effectively suppress unwanted reflections. In the reflection, interference, and transmission zone the local magnitudes of the waves can be determined for each mode by Fourier transformation at the plate surface. In simulations, both amplitudes, the normal component $v(x)$ and the in-plane component $u(x)$, can be determined. In addition, a cross section energy coefficient can be derived for each Lamb mode by Eq. (2) based on the eigenfrequency simulations for an unstructured plate. This coefficient allows to determine the wave energy in a plate cross section by evaluating the local magnitudes at the plate surface.

Tab. 1: Geometric parameters of the unit cells.

Parameter	Single-Sided UC	Double-Sided UC
B_{uc} / mm	2	2
H_{uc} / mm	1.5	1.5
B_{uc} / mm	1	1
H_{uc} / mm	0 - 0.5	0 - 0.25

$$E(f) = \frac{\int_0^{H_{\mathrm{uc}}} |E_{\mathrm{pot}}(f,h)| + |E_{\mathrm{kin}}(f,h)| \cdot \mathrm{d}h}{|u(f)|^2 + |v(f)|^2} \qquad (2)$$

These energy values can be utilized to establish an energy balance for the model. The energy of the incident wave is equivalent to the sum of the energy of all transmitted and reflected modes of the PC. Furthermore, the interference zone facilitates a comparison between the data from the reflection and transmission zones. By evaluating this model for different frequencies, the response behavior of the PC can be precisely determined.

The energy coefficients calculated by Eq. (2) are displayed in Fig. 3 for three different Lamb modes. In contrast to the A0 and S0 modes, the A1 mode exists only above a cut-off frequency of about 1 MHz. Consequently, below this frequency the incoming A0 mode can only be converted into a S0 mode by the PC, while above the S0 mode has to be considered beside A0 and S0 modes.

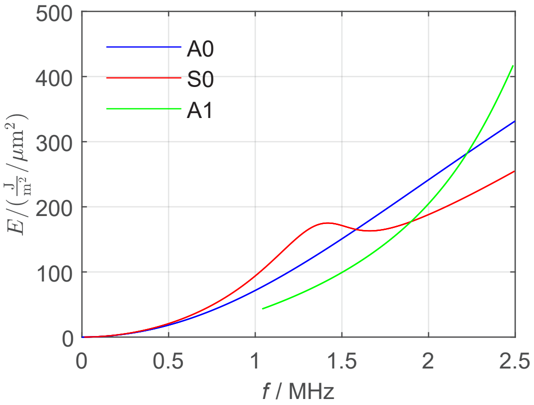

Fig. 3: Energy coefficient for squared surface elongation magnitude for different Lamb modes.

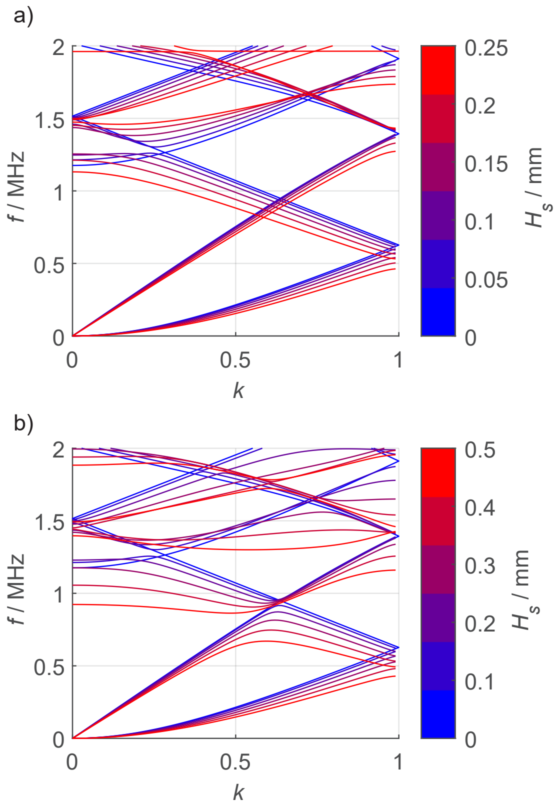

Fig. 4: Calculated band structure diagrams for a) double-sided and b) single-sided unit cells with varying parameter H_s based on a 1.5 mm aluminum plate.

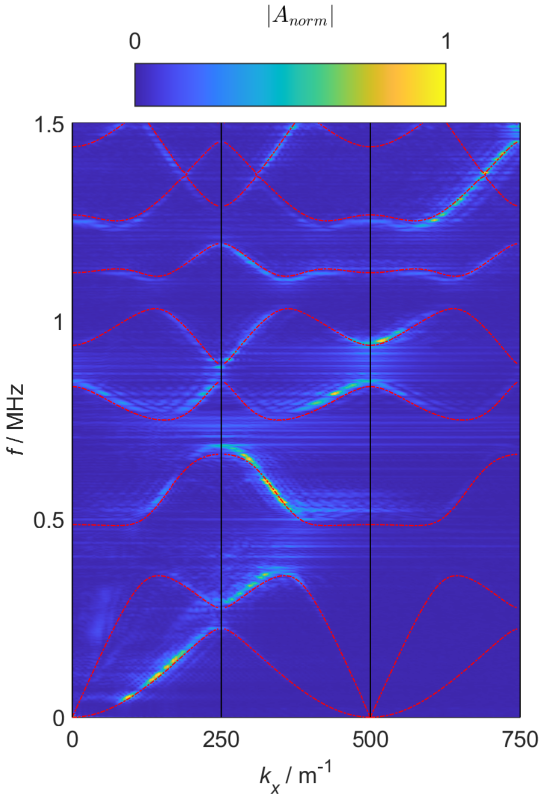

Fig. 5: Band structure measured on a 1.5 mm thick brass plate with 50 periodic milled single-sided PC (surface plot) overlaid with the simulated expanded band structure diagram (red line).

For experimental validation a single-sided structured plate with 50 milled grooves was prepared. The excitation of Lamb waves was generated by a piezoelectric transducer connected to a sine wave generator in combination with an amplifier for higher amplitudes. For characterization of the waves the local motion of the surface on the counter side of the structured area was sampled by a LDV. The evaluation of the band structure diagram was performed by 2-dimensional Fourier transformation [6].

Results

The influence of single-sided and double-sided PCs on the band structures of Lamb waves is illustrated in Fig. 4. In this graphs the depth of the structures has been gradually increased. In both diagrams, the structuring causes the mode branches to split at the edges $k = 0$ and $k = 1$. This phenomenon results in band gaps with no possible propagation inside. Furthermore, it can be observed in (Fig. 4a) that the implementation of a PC leads to uniform transitions between modes at the original intersection points between the A0 and A1 modes ($k = 0.25$, $f = 1.25$ MHz). A detailed examination using the fast Fourier transform (FFT) reveals the existence of multiple wavelengths occurring simultaneously on both branches. This phenomenon can be attributed to the transitions occurring in the diagrams. Consequently, the diagrams cannot be unfolded as for a flat plate to derive the dispersion relation. Therefore, the designations for Lamb waves, such as the A0 mode, cannot be adopted for PCs. In case of a single-sided structure (Fig. 4b), it can be observed that these interactions also occur between antisymmetric and symmetric modes ($k = 0.6$, $f = 0.9$ MHz) caused by symmetry break of the structure. This phenomenon can result in the formation of band gaps, which theoretically preclude the transmission of incident Lamb waves within specific frequency ranges.

This behavior can be confirmed in an experiment with a structured brass plate. As illustrated in Fig. 5, the measured band diagram is compared to the corresponding simulation periodically extended. As can be seen, the simulation is capable of accurately pre-

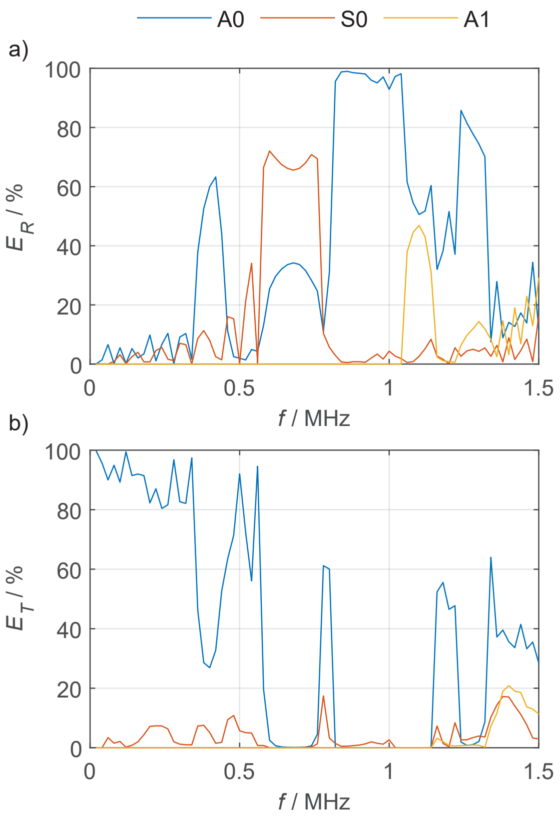

Fig. 6: Simulated relative (a) reflected and (b) transmitted energy of an inconming A0 Lamb mode by a single-sided PC of 10 elements.

dicting the band structure of PCs. As predicted, different wavelengths can be detected in the signals for the same frequencies related to the same simulated branches. However, the manner in which these crystals respond to incident waves can only be calculated from the second simulation.

Fig. 2 reveals the reflection and transmission behavior of 10 one-sided structured PCs to an incident A0 mode. This analysis is based on the band structure of Fig. 6. As assumed, low frequency A0 modes are able to pass the crystal. In the center of the displayed frequencies, an augmented reflection is observed, with the S0 mode predominating in the reflected signal. At frequencies exceeding 1 MHz, as anticipated, S0 mode components were excited by mode conversion.

Discussion and Outlook

The findings indicate that the behavior of PCs in combination with Lamb waves can be efficiently investigated through the utilization of the aforementioned methods. The simulated band structures have shown a strong correlation with the experimental data. However, it was demonstrated that a more detailed analysis of the interaction is necessary, particularly when considering finite crystals. The primary advantage of the method presented here is that, by employing suitable coefficients, an energy balance can be conducted using only the magnitudes at the surface. Consequently, the method can be applied to experiments with LDV data. However, this method requires the consideration of additional effects such as attenuation and the directivity of waves on real 3D plates. The subsequent step will involve the characterization of PCs with fluid loading, which promises to be a fascinating avenue of research. A particularly salient aspect of this study is the potential of extending the calculation to acoustic streaming, a development that promises to unlock a range of novel applications, including acoustically supported electroplating processes on structured electrodes.

Funding

This research was funded by: Technologie Allianz Oberfranken TAO; Bayerisches Staatsministerium für Wirtschaft, Landsentwicklung und Energie; Bayerisches Staatsministerium für Wissenschaft und Kunst; Bundesministerium für Bildung und Forschung and Gemeinsame Wissenschaftskonferenz GWK

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