DOI 10.5162/Ultrasonic2025/P1.6



# Optimization Design of Basic Structure of Dry Coupling Shear Wave Probe Based on Piezoelectric Double-laminated Vibrator

*Yonghao Lu*[1,2], *Yinqiu Zhou*[1,2], *Chenhui Zhu*[1], *Xueshen Cao*[1], *Hao Chen*[1,2], *Dehua Chen*[1,2], *and Zhixiang Sun*[1,2]
[1]*Institute of Acoustics, Chinese Academy of Sciences, Beijing 100190, China*
[2]*University of Chinese Academy of Sciences, Beijing 100049, China*
*zhouyinqiu@mail.ioa.ac.cn*

**Abstract:** Conventional surface-coupled probes require good contact surface flatness and efficient coupling agents, while the absence of these conditions is often encountered in field tests. In this paper, we propose a dry coupling shear wave probe based on the double-laminated vibrator. The resonant frequencies calculated by two beam theories are compared with the simulation values, and the importance of selecting the appropriate theoretical model for optimization is discussed. Simultaneously, we discuss the backing layer suitable for the small-sized low-frequency probe in this paper. Instead of the solid-state backing, we select the high-viscosity liquid damping and preliminarily verify its effect through the testing.

**Keywords:** double-laminated, dry coupling, backing layer, shear horizontal wave.

## Introduction

The ultrasonic pulse echo technique is a prevalent method in exploration engineering, valued for its simple testing procedure, cost-effectiveness, and straightforward interpretation of its results. Within this technology, shear waves have shorter wavelengths thus higher resolution than longitudinal waves at a given frequency, making them particularly effective for larger-scale structure detections. The SH0 wave mode is especially advantageous, as it is non-dispersive in the isotropic medium. Because its particle vibration is perpendicular to the wavefront, the propagation of the SH0 wave is less affected by the surrounding media [1]. Furthermore, there is less mode conversion when the SH0 wave encounters defects or boundaries [2]. These characteristics of the SH0 wave can reduce the complexity of recorded signals and can facilitate the interpretation of results.

Conventional ultrasonic testing probes are primarily surface-coupled with large flat surfaces. However, materials such as rocks and concrete often have uneven and rough surfaces, so they require complicated processes such as grinding and polishing contact surfaces before testing. Furthermore, an appropriate coupling agent must be applied to fill air gaps between the probe and the material, thereby minimizing signal loss. In some cases, surface cleanliness must be considered, and the prolonged use of numerous coupling agents is cumbersome and inconvenient. In certain scenarios, the use of liquid coupling agents is entirely prohibited. Especially for shear wave probes, high-viscosity coupling agents have the potential to make test signals unstable, easily resulting in signal loss [3]. In consequence, to address these challenges, a dry coupling technique is required for materials such as rock and concrete.

This paper focuses on a dry coupling shear wave probe designed with a piezoelectric double-laminated vibrator and a point contact structure. The vibrator is composed of two identical piezoelectric ceramic plates in a laminated structure. The flexural vibration of this double-laminated vibrator can be used to excite low-frequency shear waves within a small size of the probe for crack detection. The hemispherical shape of the dry coupling head provides point contact, which improves the ability of the probe to couple with different complex surfaces.

## Theoretical Analysis and Simulations of the Piezoelectric Double-laminated Vibrator

For porous materials such as rock and concrete, the low-frequency shear wave is the optimal detection method due to its lower acoustic attenuation. Furthermore, to enable subsequent crack detection with array imaging, it also requires a compact probe design to allow the array integration. Flexural vibration occurs at a significantly lower resonant frequency than either longitudinal vibration or thickness vibration at the same geometry, which is the most appropriate method to generate the low-frequency shear wave for these situations. The operating principle of the double-laminated vibrator is that when an electric field is applied, one piezoelectric layer extends while

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the other contracts. This opposing action causes the entire rectangular structure to produce a flexural vibration.

To get the first-order flexural vibration resonant frequency of the double-laminated vibrator, we use two distinct beam theories: the Euler-Bernoulli theory and the Timoshenko theory. The Euler-Bernoulli theory is predicated on two fundamental assumptions: (1) the cross-section remains planar post-deformation, and (2) the cross-section remains normal to the beam's axis post-deformation. This framework is primarily applicable to thin beams, with its characteristic equation for the first-order resonant frequency presented as Equation 1 [4],

$$cos(\beta L)cosh(\beta L) - 1 = 0 \quad (1)$$

where $\beta^4 = \rho S\omega^2/YI$, $\rho$ is the density of the piezoelectric ceramic, $S$ is the cross sectional area, $\omega$ is the angular frequency, $Y$ is the elastic compliance constants of the piezoelectric material, $I = SH^2/12$ is the inertia moment, and $L$ is the vibrator length.

On the other hand, the Timoshenko beam theory [5]incorporates shear strain by changing the second assumption, positing that the cross-section is no longer perpendicular to the axis after deformation, which makes it suitable for moderately thick beams. The characteristic equation of its resonance frequency is shown in Equation 2,

$$\left(\rho^2 SI\right)\omega^4 - \left[YI\rho S\beta^2 + k_s SG\left(\rho S + \rho I\beta^2\right)\right]\omega^2 + YIk_s SG\beta^4 = 0 \quad (2)$$

where $G = Y/2(1+\upsilon)$ is the shear modulus, $\upsilon$ is is the Poisson's ratio, $k_s \approx 5/6$ is the correction factor [6].

By solving both characteristic equations, we obtain sets of resonant frequencies with different lengths and thicknesses. Concurrently, we develop a 3D model of the piezoelectric double-laminated vibrator using finite element analysis software to calculate the resonant frequency of the first-order flexural vibration, comparing the simulated results with the analytical values. Fig. 1 illustrates the differences between the theoretical values derived from the Euler-Bernoulli theory and the simulation results, presenting their relative errors. Similarly, Fig. 2 displays the differences with their relative errors between the Timoshenko theoretical values from the simulations. The comparison revealed that for a small length-to-thickness ratio (L/H), the first-order resonant frequencies calculated by the Euler-Bernoulli theory are significantly higher than the simulation values, while the Timoshenko theory corrects this discrepancy effectively, with calculated frequencies closely matching the simulation values with minimal errors. Conversely, for a large L/H ratio, both theoretical results are in close agreement with simulation values; however, the Euler-Bernoulli theory has smaller errors under this condition. Notably, in Fig. 2 the simulated values are consistently higher than the theoretical values because the width is taken into consideration in the finite element software. During flexural vibration, the width is influenced by the mode shape, which provides additional stiffness to the whole structure and results in a higher simulated resonant frequency.

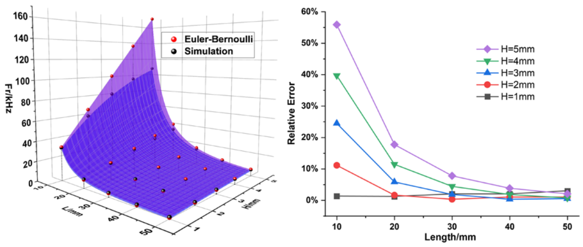


*Fig. 1: Comparison of the Euler-Bernoulli theoretical values and simulation results.*

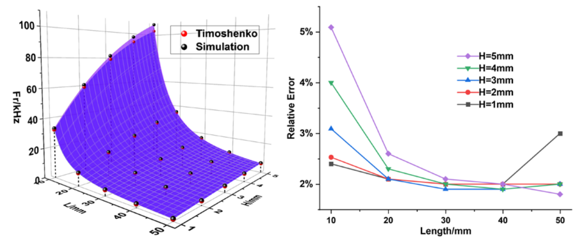


*Fig. 2: Comparison of the Timoshenko theoretical values and simulation results.*

Most piezoelectric double-laminated vibrators are designed with a high length-to-thickness ratio (L/H > 10), and for such configurations, the Euler-Bernoulli beam theory is better suitable for guiding the optimal design as it provides a simpler analytical form for resonant frequencies. However, considering the subsequent array imaging application, we require both to minimize the probe's size and to ensure the probe resolution as high as possible within a low-frequency range. Therefore, we select the operating frequency approaching 100 kHz, which corresponds to short, thick beam models. As the comparison of Fig. 1 and Fig. 2 clearly shows, we have to use the Timoshenko theory to guide our optimization in this paper and ultimately chose a model with H=4mm and L=10mm

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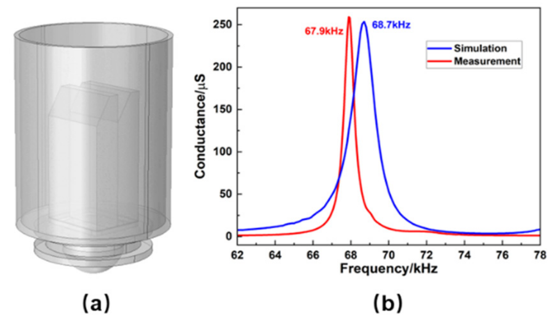


*Fig. 3: The probe basic structure with air backing. (a)physical model, (b) comparison of simulated and measured conductance.*

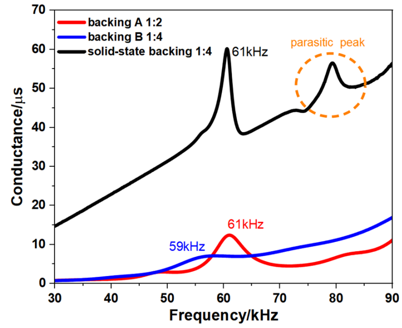


*Fig. 4: Admittance curves under different backing layers.*

as the low-frequency shear wave source, operating at approximately 97 kHz.

In addition, the general rule of the resonant frequency changing with three-dimensional sizes has been discussed in the previous study [7], where we also studied the elastic wave field generated by the double-laminated vibrator and determined that this structure has a high ratio of shear wave to longitudinal wave with the omnidirectional SH-waves. Besides, we analyzed the effect of the dry coupling head on probe energy loss.

### Backing Layer Experimental Test

After confirming that the operating frequency of the piezoelectric vibrator is consistent with the simulation result, which is the most important part of the probe, we proceed to consider other structures. This involves bonding a wear-resistant ceramic tip to the vibrator to serve as a dry coupling head and adding a backing layer between the vibrator and the probe shell. Firstly, we calculate the admittance curve of this basic structure without a backing layer, that is, the air backing condition, and compare it with the measured result. Furthermore, the basic structure of the probe, including the optimized piezoelectric vibrator, probe shell, and the dry coupling head, is constructed using finite element simulation software. The resonant frequencies obtained from the simulation are compared with the experimentally measured frequencies in Fig. 3b, which are in close agreement. The operating frequency is changed to 68 kHz as a result of the added mass and stiffness introduced by the dry coupling head and the shell.

Since the size of the vibrator is only 10mm in length and the diameter of the basic structure of the probe is only about 15mm, the wavelength is close to or even higher than the geometric size. In this case, the backing layer is too small to effectively attenuate the sound wave by internal propagation. Furthermore, a conventional solid backing made of the epoxy-tungsten powder composite not only provides insufficient damping, but its added mass and stiffness can also disrupt the vibrator's natural resonance, which may lead to the parasitic resonance peak near the primary ones. For this reason, we consider using a liquid-state epoxy-tungsten powder mixture as a high-efficiency viscous damping system with the weak stiffness effect but the prominent viscous energy consumption, which facilitates the rapid attenuation of the vibration.

Therefore, we prepare three different backing layer samples. The backing A is formulated with a 1:2 mass ratio of epoxy resin to tungsten powder while the backing B uses a 1:4 ratio. Both are prepared without the curing agent and finally presented as a highly viscous liquid state. For comparison, a solid-state backing is created using the same formulation as Backing B but with the curing agent added.

The admittance curves in Fig. 4 clearly show the parasitic resonance peaks caused by the solid backing, while the two liquid-state backings demonstrate the ability to effectively suppress this parasitic peak. Due to the influence of additional mass, the heavier backing B has a lower resonant frequency than the backing A, which are respectively 59 kHz and 61 kHz. The solid-state backing is affected by both additional mass and stiffness, and the main resonance peak is located at 61 kHz. To analyze the damping effect of the backing, the time-domain pulse signals of the vibrator under different backings are shown in Fig. 5. The signal of the probe with backing B, which has a higher damping, attenuates faster than that with backing A. Also, the signal of the probe with solid-state backing attenuates less than the liquid-state backing under the same mass ratio as expected.

### Conclusion and Discussion

Based on rectangular piezoelectric double-laminated models, finite element simulations are conducted to optimize the size and shape of the vibrator. The gen-

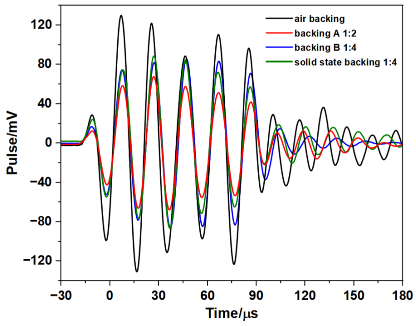


*Fig. 5: Pulse ringing attenuation under different backing layers.*

eral rule is that the resonant frequency of flexural vibration has a negative correlation with the length, while it has a positive correlation with the thickness of the vibrator. For structures of varying dimensions, the selection between the Euler-Bernoulli and Timoshenko theories should be predicated on the length-to-thickness ratio (L/H) to calculate the analytical solution for flexural vibration. Through design optimization, we eventually determine the appropriate structure and dimensions of the double-laminated vibrator. The basic structure prepared based on optimization results shows excellent agreement with simulations in both its admittance curve and resonant frequency. Moreover, after the theoretical analysis of the backing layer of the small-sized probe, we propose the structure of liquid damping and verify that is better than the solid backing in this probe after the preliminary test. With the study proceeding, we will study further and optimize the design of backing layer, ultimately completing the experimental verification of the probe.

## Acknowledgment

This work is supported by the National Natural Science Foundation of China (Grant Nos. 42127807,12274432, 52227901).

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