DOI 10.5162/iCCC2026/7.3

Ultrasound-Based Interferometric Hip Implant Monitoring: Analysis of Frequency Selection and Minimum Detectable Gap Width

Jan Lützelberger and Klaus Stefan Drese
ISAT – Institute of Sensor and Actuator Technology, Coburg University of Applied Sciences and Arts, Coburg, Germany
jan.luetzelberger@hs-coburg.de

Abstract

Early detection of aseptic or septic loosening of hip endoprostheses is clinically crucial but still relies primarily on projection radiography, which lacks sensitivity for sub-millimetre interlayers at the bone–implant interface. Recent work by Lützelberger et al. demonstrated that ultrasound interferometry enables quantification of the width of this interface by analysing frequency-domain interference patterns of reflections from the soft-tissue gap. However, several open theoretical questions remained, particularly regarding the fundamental detection limits for small gap widths and the optimal choice of excitation frequency.

This work provides a mathematical analysis of (i) the minimal resolvable gap width based on the Gabor limit, bandwidth constraints, and the signal-to-noise ratio (SNR) and (ii) the optimal operating frequency considering attenuation in soft tissue and the limited thickness of the cortical bone layer. The results show that an excitation frequency around 3 MHz represents an optimum here. Furthermore, closed-form expressions for the minimum detectable width are derived, predicting detection thresholds in the range of 156–212 µm for different SNR conditions – consistent with previously reported experimental results. These findings provide theoretical justification for system design choices in interferometric ultrasound for implant-integration monitoring.

Kurzfassung

Die Früherkennung einer aseptischen oder septischen Lockerung von Hüftendoprothesen ist klinisch von entscheidender Bedeutung, stützt sich jedoch nach wie vor in erster Linie auf Projektionsradiographie, die für submillimetergroße Zwischenräume an der Schnittstelle zwischen Knochen und Implantat nicht empfindlich genug ist. Jüngste Arbeiten von Lützelberger et al. haben gezeigt, dass die Ultraschallinterferometrie eine Quantifizierung der Breite dieser Schnittstelle durch Analyse der Interferenzmuster im Frequenzbereich der Reflexionen aus dem Weichteilspalt ermöglicht. Es blieben jedoch einige theoretische Fragen offen, insbesondere hinsichtlich der grundlegenden Nachweisgrenzen für kleine Spaltbreiten und der optimalen Wahl der Anregungsfrequenz.

Diese Arbeit liefert eine mathematische Analyse (i) der minimalen auflösbaren Spaltbreite auf Grundlage der Gabor-Unschärfe, der Bandbreitenbeschränkung des Ultraschallpulses und des Signal-Rausch-Verhältnisses (SNR) sowie (ii) der optimalen Sendefrequenz unter Berücksichtigung der Dämpfung im Weichgewebe und beschränkten Dicke des kortikalen Knochens. Die Ergebnisse zeigen, dass eine Anregungsfrequenz um 3 MHz hier einen optimalen Kompromiss darstellt. Darüber hinaus werden geschlossene Ausdrücke für die minimal erkennbare Spaltbreite abgeleitet, die Erkennungsschwellen im Bereich von 156–212 µm für unterschiedliche SNR-Bedingungen vorhersagen – in Übereinstimmung mit zuvor vorgestellten experimentellen Ergebnissen. Diese Ergebnisse liefern eine theoretische Begründung für Systemdesignentscheidungen bei interferometrischem Ultraschall zur Überwachung der Implantat-Integration.

1 Introduction

1.1 Clinical Motivation

Hip joint prostheses are among the most frequently implanted orthopedic devices, yet aseptic loosening continues to be the most common long-term complication [1]. Loosening originates from the formation of a soft-tissue layer between bone and implant – typically in the range of a few hundred micrometers to several millimeters [2]. Detecting early-stage changes in the gap width is clinically important, both for detecting loosening itself and for identifying tissue changes caused by bacterial biofilms [3]. Current diagnostics rely largely on plain radiography, which lacks the spatial resolution needed to detect gaps below 1 mm and cannot characterize the interfacial tissue [4]. Ultrasound offers a radiation-free alternative, but classical time-of-flight ultrasound imaging fundamentally struggles with thin interlayers: when the gap width is about or smaller than the acoustic wavelength, reflections from the front and back surfaces overlap in time and the interlayer thickness as a correlate to loosening cannot be determined directly in time domain [5,6,7].

1.2 Interferometric Ultrasound Approach

Lützelberger et al. [8] proposed a frequency-domain approach using the interferometric pattern generated by multiple reflections within the bone–gap–implant three-layer system. Using different idealized setups as well as a more realistic bone-implant system, the method could successfully determine water gap widths in the range of approx. 160 µm to 2 mm. While experimentally successful, the

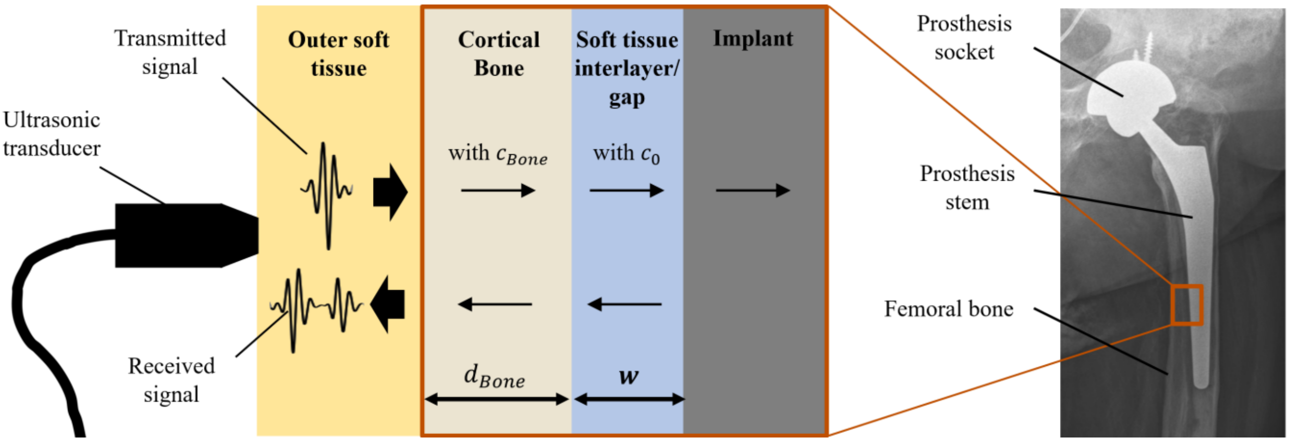

Figure 1 Multilayer model of the thigh with hip prothesis stem.

method still lacks a rigorous theoretical justification for two critical design choices:

1. What fundamental limit exists for the minimum detectable gap width, and how this limit depends on pulse length and SNR.
2. Why an excitation frequency of approximately 3 MHz is reasonable, given large anatomical variability and strong frequency-dependent attenuation.

The aim of this paper is therefore to provide a compact analytical treatment of these questions, suitable for guiding future clinical device optimization.

2 Methods

2.1 Simplified Physical Model of Ultrasonic Reflection

As Figure 1 shows, the anatomical situation of a thigh bone with a hip prosthesis stem is modelled as a layered system consisting of soft tissue (skin, fat, muscles), cortical bone, a thin soft-tissue interlayer of width w, and the implant [8,9]. For longitudinal plane waves, multiple reflections at the layer interfaces interfere, forming a frequency-dependent reflection coefficient $R(f)$ [8,10].

For gaps small compared to the wavelength, time-domain separation of the frontside and backside reflections is impossible. Instead, destructive interference produces periodic minima in the amplitude spectrum of the reflected signal. Their spectral position f_s follows from [11]

$$f_s = s\frac{c_0}{2w} \tag{1}$$

with $s \in \mathbb{Z}$ where c_0 is the sound velocity in the interlayer. The spacing of the minima therefore results from [12]

$$\delta f = \frac{c_0}{2w}. \tag{2}$$

Thus, the measurement task for determining the interlayer width w reduces to detecting the position f_s of minima within the amplitude spectrum or their spacing δf.

2.2 Bandwidth Constraints from the Gabor Limit

For time-frequency analyses in signal processing, uncertainty can be expressed by the Gabor limit [13] stating

$$\sigma_t \sigma_f \geq \frac{1}{2\pi} \tag{3}$$

for the relation between the standard deviations σ of time t and frequency f, referring to amplitude signals. For a Gaussian-shaped pulse, using the Full Width at Half Maximum (FWHM) Δ with

$$\Delta = 2\sqrt{2\ln(2)}\ \sigma \tag{4}$$

as the characteristic pulse width is more suitable because it is most straight forwardly determined from experimental data. This yields to

$$\Delta_t \Delta_f \geq \frac{4\ln(2)}{\pi} \approx 0.88. \tag{5}$$

Consequently, shorter pulses provide larger bandwidth, which increases the number of observable minima.

2.3 SNR-Dependent Detectable Gap Width

Assuming a Gaussian shaped amplitude spectrum $A(f)$ of the excitation signal (see Figure 2) with

$$A(f) = A_0\, \mathrm{e}^{-\frac{1}{2}\left(\frac{f-f_0}{\sigma_f}\right)^2} \tag{6}$$

with central frequency f_0 and maximal amplitude A_0 is a reasonable approximation for the Hanning windowed sine burst used by Lützelberger et al. [8].

To detect at least one spectral minimum in the presence of white noise with amplitude level N_0, the minimum position f_s needs to be within the interval

$$f_{N_0,-} < f_s = \frac{c_0}{2w} < f_{N_0,+}\,, \tag{7}$$

using (1), whereby $f_{N_0,-}$ and $f_{N_0,+}$ are the limits of the usable bandwidth of the transducer, i.e., the frequencies where the noise level N_0 crosses the spectrum $A(f)$ of the excitation signal.

Rearranging (7) with respect to w leads to the limits of detection of the interlayer width with

$$w < \frac{s c_0}{2 f_{N_0,-}} \tag{8}$$

as the upper limit of the detectable gap width w as well as

$$w > \frac{s c_0}{2 f_{N_0,+}} \tag{9}$$

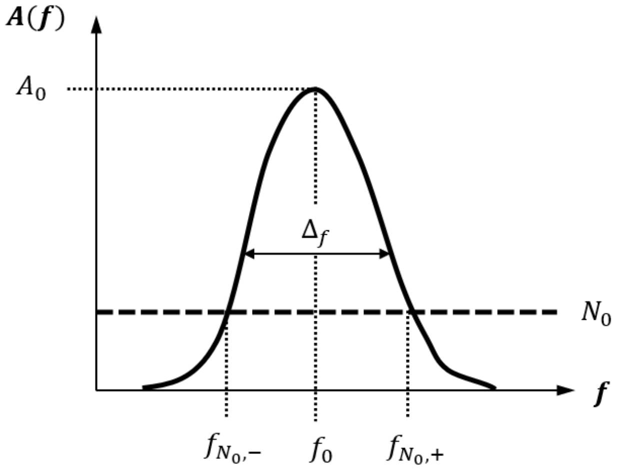

Figure 2 Schematic drawing of the amplitude spectrum $A(f)$ of the excitation signal in [8] including a white noise level N_0.

as the lower limit of the detectable gap width w.

$f_{N_0,-}$ and $f_{N_0,+}$ can be obtained using (6) and (4) leading to

$$A(f_{N_0}) = A_0\,\mathrm{e}^{-\frac{4(f_{N_0}-f_0)^2\ln(2)}{\Delta_f^2}} = N_0. \qquad (10)$$

Introducing the well-known signal-to-noise ratio SNR with

$$SNR = \frac{A_0}{N_0} \qquad (11)$$

and solving (10) for f_{N_0}, using (5), gives

$$f_{N_0} = f_0 \pm \frac{2\sqrt{\ln(SNR)\ln(2)}}{\pi\Delta_t} \qquad (12)$$

for $f_{N_0,-}$ and $f_{N_0,+}$, taking a given SNR und pulse width Δ_t. As large gap widths, when pulses are separated in time domain, can be resolved by a simple time-of-flight measurement [14], the primary interest lies in the lower limit of w. Inserting (12) into (8) gives

$$w_{min} = \frac{sc_0}{2f_0 + \frac{4\sqrt{\ln(SNR)\ln(2)}}{\pi\Delta_t}}. \qquad (13)$$

This provides a closed form estimate for the theoretical detection limit w_{min} of the interlayer width between bone and implant, when using (1) for width calculation.

2.4 Optimal Excitation Frequency

For a plain wave travelling through tissue of thickness l, the intensity reduction is [7]

$$I = I_0\,\mathrm{e}^{-\alpha(f)l} \qquad (14)$$

introducing the frequency dependent attenuation factor $\alpha(f)$. This factor usually follows an empirical power law

$$\alpha(f) = \alpha f^q \qquad (15)$$

with $q \approx 1$ for most technical and biological materials [7]. Consequently, aiming for the highest signal amplitude, one should reduce the transducer frequency f as far as possible. However, in the considered measurement task of a layered medium with limited thickness of each layer, there is a lower bound for the usable frequency. While pulse overlapping within the soft-tissue interlayer is no problem for measuring the interlayer width due to the spectral minima approach proposed by Lützelberger et al. [8], the limited thickness of the cortical bone layer is a critical issue: As soon as the travel time t_{Bone} within the bone layer gets smaller than the pulse duration, multiple frontside and backside reflections within the bone layer start to overlap and (1) is not applicable anymore for calculating the soft tissue interlayer width. Consequently, there is the criterion

$$t_{Bone} = \frac{2d_{Bone}}{c_{Bone}} > \frac{\beta}{f} \qquad (16)$$

with the thickness d_{Bone} of and speed of sound c_{Bone} within the bone layer as well as the number β of oscillations performed by the transducer used.

(16) as a lower frequency bound always has to be fulfilled for enabling correct interlayer width determination.

According to (14), (15) and (16), the theoretically optimal excitation frequency is the lowest frequency enabling time-domain pulse separation within the cortical bone layer, i.e., fulfilling (16). This leads to

$$f_{opt} = \frac{\beta c_{Bone}}{2d_{Bone}} \qquad (17)$$

as the optimal excitation frequency.

3 Results

3.1 Minimum Detectable Gap Width

Using realistic parameters from Lützelberger et al. [8]:

- Sound velocity: $c_0 = 1460$ m/s
- FWHM pulse width: $\Delta_t = 1.0$ µs

the theoretical detection limits for selected values of SNR, calculated with (11), become:

- For $SNR = 20000$: $w_{min} = 156$ µm
- For $SNR = 200$: $w_{min} = 173$ µm
- For $SNR = 2$: $w_{min} = 212$ µm

These values match the experimentally observed detection capabilities reported by Lützelberger et al. [8], who achieved measurable gap widths down to ≈ 160 µm under idealized conditions and gap widths down to ≈ 210 µm under more realistic conditions.

3.2 Optimal Excitation Frequency

Regarding the future clinical application, the smallest possible bone layer thickness $d_{Bone,min}$ as well as the largest possible sound velocity $c_{Bone.max}$ must be considered when calculating the optimal excitation frequency by (17). Using typical anatomical values

- Minimal bone thickness $d_{Bone,min} = 3.3$ mm [15]
- Maximal sound speed $c_{Bone.max} = 4600$ m/s [16]

and $\beta \approx 4$ considering the experimental findings by Lützelberger et al. [8], one obtains

$$f_{opt} \approx 2.79 \text{ MHz}.$$

This aligns with the experimental choice and confirms the suitability of the 3 MHz probe used in [8].

4 Discussion

The detection-limit analysis shows that the minimal resolvable gap width depends primarily on:

1. Pulse duration Δ_t, governed by transducer physics.
2. SNR, but only logarithmically – meaning that improving SNR beyond moderate values yields diminishing returns.
3. Sound speed and anatomical conditions, which are fixed and patient dependent.

Importantly, the theoretical minimum gap widths derived here (156 – 212 µm) correspond closely to those experimentally measured values of Lützelberger et al. [8]. This agreement indicates that the interferometric method is operating near the physically achievable limits imposed by acoustic uncertainty relations.

Furthermore, the analysis demonstrates that the 3 MHz excitation frequency in the experiments of Lützelberger et al. [8] is near the analytically derived lower frequency limit governed by the limited bone thickness. It represents a good choice as it reduces frequency-dependent tissue attenuation as far as possible.

The results also highlight the fundamental advantage of frequency-domain analysis over time-of-flight methods, which fail entirely for gaps smaller than the wavelength.

5 Conclusion

This study provides a theoretical foundation for two central design choices in interferometric ultrasound for hip implant loosening diagnostics:

- The minimum detectable gap width is constrained by pulse bandwidth and SNR and lies at approx. 156–212 µm for various measurement conditions.
- The excitation frequency of 3 MHz arises from optimizing the trade-off between attenuation and avoiding pulse interference within the cortical bone layer.

These findings match experimental observations and support the continued development of frequency-domain ultrasound for early loosening detection and implant integration monitoring. They also offer guidance for future transducer and system design, particularly regarding pulse shaping and optimal frequency selection.

Future investigations include a detailed exploration of the interactions between minimum detectable gap width and optimal excitation frequency and investigation of further system design aspects, e.g., the transducer assembly, sound field characteristics and non-perpendicular excitation.

6 Literature

[1] Jennison, T.; MacGregor, A.; Goldberg, A.: Hip arthroplasty practice across the organisation for economic co-operation and development (OECD) over the last decade. The Annals of The Royal College of Surgeons of England. Vol. 105 (2023), no. 7, pp. 645-652.

[2] Apostu, D.; Lucaciu, O.; Berce, C., Lucaciu, D.; Cosma, D.: Current methods of preventing aseptic loosening and improving osseointegration of titanium implants in cementless total hip arthroplasty: a review. Journal of International Medical Research. Vol. 46 (2018), no. 6, pp. 2104-2119.

[3] Wirtz, D.C.; Niethard, F.U.: Ursachen, Diagnostik und Therapie der aseptischen Hüftprothesenlockerung – Eine Standortbestimmung. Zeitschrift für Orthopädie und Unfallchirurgie. Vol. 135 (1997), no. 4, pp. 270-280.

[4] Ruther, C.; Timm, U.; Ewald, H.; Mittelmeier, W.; Bader, R.; Schmelter, R.; Lohrengel, A., Kluess, D.: Current possibilities for detection of loosening of total hip replacements and how intelligent implants could improve diagnostic accuracy. Recent Advances in Arthroplasty. InTech. (2012), pp. 363-386.

[5] Lewin, P.A.: Quo vadis medical ultrasound? Ultrasonics. Vol. 42 (2004), no. 1-9, pp. 1-7.

[6] Powers, J.; Kremkau, F.: Medical ultrasound systems. Interface Focus. Vol. 1 (2011), no. 4, pp. 477-489.

[7] Wells, P.N.T. (Ed.): Ultraschall in der medizinischen Diagnostik. Berlin und Boston: De Gruyter, 2019.

[8] Lützelberger, J.; Arneth, P.; Franck, A.; Drese, K.S.: Ultrasonic Interferometric Procedure for Quantifying the Bone–Implant Interface. Sensors. Vol. 23 (2023), no. 13, pp. 5942.

[9] Puleo, D.: Understanding and controlling the bone–implant interface. Biomaterials. Vol. 20 (1999), no. 23-24, pp. 2311-2321.

[10] Reddyhoff, T.; Kasolang, S.; Dwyer-Joyce, R.S., Drinkwater, B.W. The Phase Shift of an Ultrasonic Pulse at an Oil Layer and Determination of Film Thickness. Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology. Vol. 219 (2005), no. 6, pp. 387-400.

[11] Pialucha, T.; Guyott, C.C.H.; Cawley, P.: Amplitude spectrum method for the measurement of phase velocity. Ultrasonics. Vol. 27 (1989), no. 5, pp. 270-279.

[12] Kinra, V.K.; Jaminet, P.T.; Zhu, C.; Iyer, V.R.: Simultaneous measurement of the acoustical properties of a thin-layered medium: The inverse problem. The Journal of the Acoustical Society of America. Vol. 95 (1994), no. 6, pp. 3059-3074.

[13] Mallat, S.: A wavelet tour of signal processing. San Diego, London: Academic Press – An Imprint of Elsevier, 1999.

[14] Kinra, V.K.; Iyer, V.R.: Ultrasonic measurement of the thickness, phase velocity, density or attenuation of a thin-viscoelastic plate. Part I: the forward problem. Ultrasonics. Vol. 33 (1995), no. 2, pp. 95-109.

[15] Croker, S.L.; Clement, J.G.; Donlon, D.: A comparison of cortical bone thickness in the femoral midshaft of humans and two non-human mammals. Homo. Vol. 60 (2009), no. 6, pp. 551-565.

[16] Salehi, L.: Nonlinear tomographic reconstruction of elastic properties of isotropic solid materials from ultrasound measurements. Ph.D. Thesis, Ruhr Universität, Bochum, Germany, 2016, p. 5. Available online: http://hss-opus.ub.ruhr-unibochum.de/opus4/frontdoor/index/index/docId/5208.