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# Optimal Design of Cantilever-type Vibro Harvester Working at Second Eigenfrequency

*Paulius Skėrys*[1], *Dr. Rimvydas Gaidys*[1], *Dr. Birutė Narijauskaitė*[1]
[1] *Kaunas University of Technology, Studentų g. 56-344, Kaunas, Lithuania*
*paulius.skerys@ktu.lt*

## Summary:

The research presents an optimization procedure for the thickness of the active element in a piezoelectric energy harvester for the fixed second eigenfrequency to enhance electrical output. The cantilever beam is modelled as a two degree of freedom Bernoulli-Euler element. The optimal design cantilever achieves a 70% efficiency of generated strain compared to the uniform cantilever, with experimental results confirming the mathematical model.

**Keywords:** shape optimization, sensitivity analysis, vibration energy harvesting.

## Background, Motivation, and Objective

Recent advancements in energy harvesting technologies have garnered significant attention, particularly in piezoelectric energy harvesters (PEHs) [1]. A notable challenge in enhancing the efficiency of PEHs is the strain distribution in cantilever designs. While much of the existing beam shape optimization focuses on width [2-4], this study introduces an approach that optimizes the thickness of piezoelectric cantilevers for fixed second eigen frequency, allowing for strain maximization at the upper layer of the active element.

## Methodology

When a piezoelectric material is mechanically strained, electric polarization that is proportional to the applied strain is produced [5]. This means that if the strain is maximized, the electric polarization will be maximum too.

The optimization problem is to maximize the strain of the upper layer of the cantilever (Fig. 1) by varying its thickness for fixed second eigenfrequency. The design variable was the thickness of the cantilever, state variable - eigenfrequency.

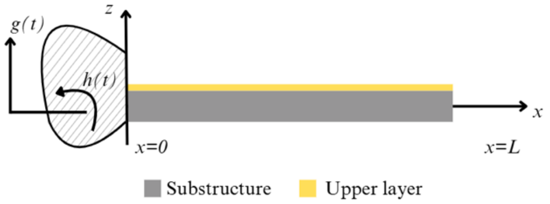

*Fig. 1. Active element of the cantilever type vibration energy harvester*

Optimization problem now can be formulated: maximize the objective function

$$Max\,\Psi_0(b)=\sum_{i=1}^{m}\left|-\frac{b_i}{2}\frac{d^2\mathrm{v}(x)}{dx^2}\right|=\sum_{i=1}^{m}|[B_i]\{u_i^e\}|$$
$$=\sum_{i=1}^{m}\left|-\frac{b_i}{2}\left[-\frac{6}{l^2}+\frac{12x}{l^3}\quad \frac{6}{l^2}-\frac{12x}{l^3}\quad -\frac{4}{l}+\frac{6x}{l^2}\quad -\frac{2}{l}+\frac{6x}{l^2}\right]\begin{Bmatrix}v_1^i\\ \theta_1^i\\ v_2^i\\ \theta_2^i\end{Bmatrix}\right| \tag{1}$$

subject to the state equation

$$K(b)\varphi=\zeta M(b)\varphi \tag{2}$$

and constraints

$$\Psi(\zeta,b)\geq 0. \tag{3}$$

Where: $i$ – finite element number, $v(\mathrm{x})$ – bending beam shape function vector, $\{u_i^e\}$ – element nodal displacement vector, $[B_i]$ - the strain-displacement matrix, $l$ – finite element length (the same for all $i=1,m$), $b_i$ – thickness of element, $K(b)$ - is the stiffness matrix, $M(b)$ - is the inertia matrix, $\varphi$ - is the eigenvector of a given vibration mode, $\zeta$ is the corresponding eigenvalue.

The gradient projection method with sensitivity analysis was used to solve this optimization problem. Initial geometry was a uniform cantilever with a second bending eigen frequency equal to the target value.

## Results

Design variables, the thickness of the beam, were constrained to minimum thickness. The state variable, second eigen frequency, was constrained to be the same ($w_2=267\,Hz$), as of the initial uniform cantilever. Optimization was simulated in MATLAB®, optimal shape cantilever was then imported into COMSOL Multiphysics ® for transient analysis.

The optimal design cantilever is presented in Fig. 2. Fig. 3 depicts a bending second eigenform of the cantilever with a normalized normal strain field. Some region of maximum strain occurs at the clamped end of the cantilever, however, a second range of strain is observed in the middle section. The third part of the cantilever exhibits negligible strain and is an inertial mass.

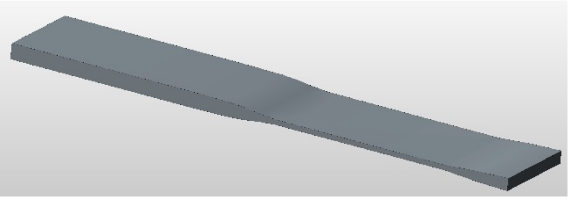

*Fig. 2. Optimal shape cantilever for the fixed second eigenfrequency*

Normal strain distribution along the cantilever during transient analysis is presented in Fig. 4. The total strain amount of the optimal design cantilever was around 70% higher compared to the uniform cantilever.

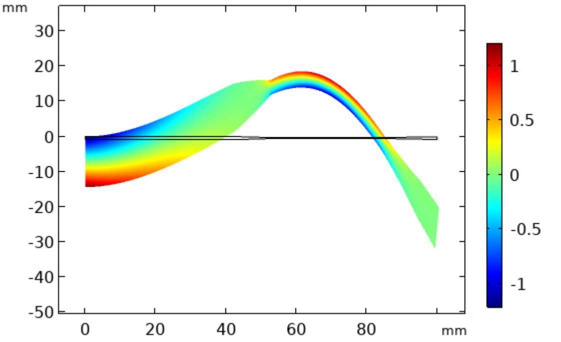

*Fig. 3. Second bending eigenform of the optimal design cantilever. Normalized normal strain field.*

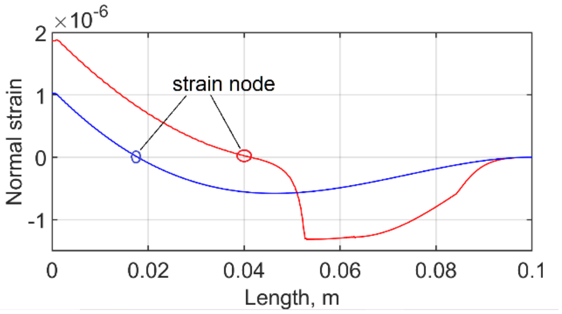

*Fig. 4. Normal strain distribution at the upper layer of the cantilever, kinematically excited by second eigen frequency $w_2 = 267\ Hz$, during transient analysis at the time moment of the max strain. Blue - uniform cantilever, red – optimal design cantilever.*

Simulation results were verified experimentally. Specimens were manufactured using additive manufacturing technology with photopolymer Tough 2000 and featured a PVDF DT1-028K/L piezoelectric sensors to generate electrical signals which were recorded by a digital oscillometer while the cantilever was excited by a vibrational plate to its second eigen frequency. In Fig. 5 the result of the experiment is presented. Maximum peak-to-peak values were compared of uniform and optimal design cantilevers.

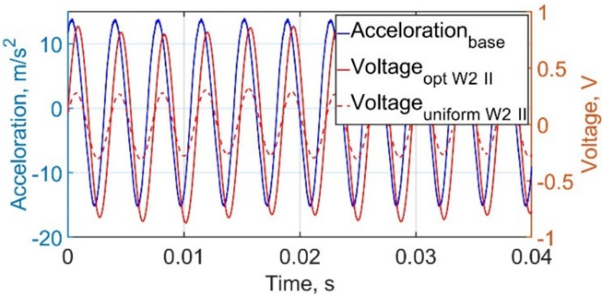

*Fig. 5. Experimental results graph. Blue line - accelerometer signal, red solid – optimal cantilevers output, red dashed – initial cantilevers output.*

In Fig. 5, the cantilevers were excited by a 14 m/s$^2$ acceleration. The uniform cantilever output is a 0.31V, optimal design - 0.8V, in cantilever's interval from strain node to tip.

## Conclusion

The optimal design cantilever for the second eigenfrequency was found, which generates a 70% more normal strain amount along the cantilever length with the fixed eigenfrequency. The simulation results were validated experimentally, confirming the generation of normal strain.

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