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Sensor Density Optimization of Active Magnetic Shielding System based on Monte Carlo Simulation

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Summary:
Active Magnetic Shielding (AMS) systems have become a key component in ensuring near-zero magnetic environments. However, there is still a lack of clear mechanisms for feedback sensor density, layout, and compensation effects in the region of interest (ROI). In this article, the Tikhonov regularization compensation algorithm is employed to analyse the dependency by Monte Carlo simulation. Results indicate a declining compensation error in any layout conditions as the sensor density increases. Therefore a sensor density optimization method based on the threshold compensation error is proposed.

Keywords: Active Magnetic Shielding, Sensor Network, Monte Carlo Simulation, Tikhonov Regularization, Statistical Analysis

Introduction

AMS systems, employing perimeter sensor networks and triaxial coils, play a critical role in ultra-low-field environments for precision experiments such as neutron electric dipole moment (nEDM) measurements and magnetoencephalography (MEG)[1,2]. Conventional approaches either address sensor placement through system ill-conditioning analysis or co-optimize layouts with compensation targets under restrictive zero-field compensation target constraints at sensor locations, faceing inherent limitations due to the coupled optimization[3,4]. This work resolves these limitations by introducing a probabilistic Monte Carlo (MC) framework, establishing quantitative relationships between sensor density, spatial distribution, and shielding efficacy, which can guide the optimization of sensor density in AMS systems.

Analysis Object

Our Investigation is based on an AMS configuration comprising a triaxial Merritt coils (four independently controlled windings per axis, 2.0 m × 1.8 m × 1.6 m) and a fluxgate magnetometer network (2-10 sensors) deployed between coil boundaries and a 5×5×5 grid ROI, which satisfies the assumption that excitation current and magnetic field response in region of interest ROI of coil exhibiting a linear or approximately linear relationship $\boldsymbol{B} = A\boldsymbol{I}$, corresponding to coil constant matrix A.

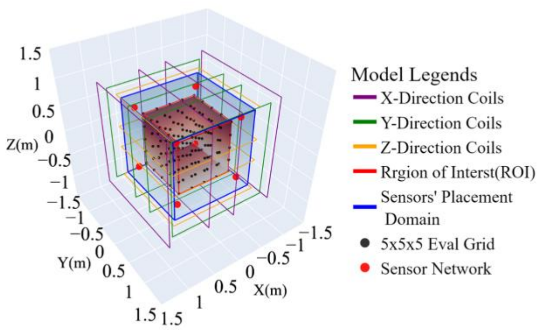

Fig. 1. Relative positions of sensor network and ROI evaluation points in AMS system

Tikhonov Regularization Compensation

A fluctuation term is introduced to the Tikhonov regularization compensation (TRC) cost function J(Δ**I**) to overcome the compensation instability in linear feedback caused by ill-posed problems of response matrix A correspond to sensor layout and coil structrue:

$$J\left(\Delta \boldsymbol{I}\right) := \frac{1}{2} \| A\Delta\boldsymbol{I} - B \|^2 + \frac{\alpha}{2} \| \Delta\boldsymbol{I} \|^2 \qquad (1)$$

Together, the linear compensation algorithom is shown as follows:

$$\Delta \boldsymbol{I}_r = A_r^{+}\boldsymbol{B} \quad (2)$$

$$A_r^{+} = V\Sigma_r^{-1}U^T \quad (3)$$

where $A = U\Sigma V^T, \Sigma_r^{-1} = diag\left(\frac{\sigma_{ii}}{\sigma_{ii}^2 + \alpha^2}\right)$.

In following MC simulations, for each different layout, an optimal transfer regularization parameter α for balancing compensation error and resolving fluctuations through grid search method is applied to ensure that TRC works in the best condition.

Density-compensation analysis based on Monte Carlo Simulation

Operating in an inhomogeneous magnetic environment $\mathbf{B}_{init}$ and $\mathbf{B}_{ext}$, the methodology integrates Monte Carlo sampling of 10,000 coil current vectors **I** and sensor layouts P with Tikhonov-regularized compensation (TRC) loops. E_{vec} and E_{sca} quantify vector and scalar relative errors between residuals across the ROI and field distribution generated from **I**, $\mathbf{B}_{init}$, $\mathbf{B}_{ext}$ after 10 control iterations under each layout P. The simulation process are as follows:

(1) Generate background magnetic field at sensors $\mathbf{B}_{init}$ and at ROI evaluation points $\mathbf{B}_{ext}$;
(2) Uniformly sampling generate different sensor layouts P and final control state defined by steady-state current **I**;
(3) Uniformly sampling 5 × 5 × 5 evaluation point grid within ROI for characterizing its magnetic field distribution and calculate:
Operating point of TRC : $\boldsymbol{B}_{set} = \boldsymbol{B}_{init} + A_P\boldsymbol{I}$

Ideal distribution in ROI : $\boldsymbol{B}_{ideal} = \boldsymbol{B}_{ext} + A_{ROI}\boldsymbol{I}$;
(4) The TRC is driven at each sensor layout P in 10 iterations of compensation, thereby computing the set of scalar and vector deviations, E_{sca} and E_{vec} between the magnetic field distribution $\mathbf{B}_{comp}$ within the ROI at steady state and $\mathbf{B}_{ideal}$;
(5) Repeat simulation for all sensor density N.

Results and Conclusions

Run MC simulation for N=2~10 under 100-sized sampled **P** and **I** in each turn: As sensor density N increases, 95% percentile of errors E_{vec} declines from 369.17% to 8.49%, while for E_{sca} is 55.1% to 1.77%, revealing a more and more significant statistical decoupling between sensor layout and TRC compensation performance in ROI.

The optimization framework can be formally described as follows. First, compute the 95th percentile $Q_{0.95}(E)$,(E= E_{vec}, or E_{sca}) across all simulation scenarios to establish a conservative upper bound for the probability estimation error.

Subsequently, determine the minimum sensor density N that satisfies the constraint where error ``metric remains within the predefined maximum tolerance threshold E_{max}. This constrained optimization problem can be mathematically formulated as:

$$\begin{cases} N = argmin\left(Q_{0.95}(E)\right) \\ s.t. E < E_{\max} \end{cases} \quad (4)$$

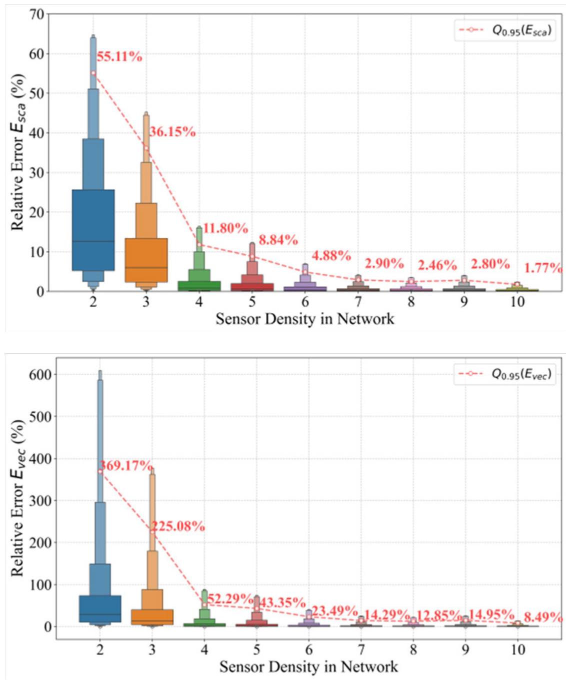

Fig. 2. Distribution and 95% percentile value of E_{sca} and E_{vec}.

In addition, with proper sensor density, the sensor layout and compensation performance can be successively optimized independently, where the latter has a potential improvement by tuning the AMS operate points via operating points based on target field method.

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