

Geometrical Optimization of Planar Hall Sensors Dominated by $1/f$ Noise for Maximal Signal-to-Noise Ratio

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Summary:

This work introduces a voltage-related geometry factor for arbitrarily shaped planar Hall devices dominated by $1/f$ noise and derives a corresponding signal-to-noise (SNR) geometry factor. The geometry factors are simulated for disk-, square-, and cross-shaped Hall sensors. The results are supported by measurements on cross-shaped GaAs Hall sensors and confirm a trade-off between Hall signal and noise performance. Optimal cross geometries provide the highest SNR geometry factor, namely 0.59, while optimal disk and square geometries reach only 0.56 and 0.52, respectively.

Keywords: Hall sensor, signal-to-noise ratio (SNR), $1/f$ noise, optimization, geometry factor

Introduction

The signal-to-noise ratio (SNR) is a key figure of merit for Hall sensors. It is defined as the ratio of Hall signal to noise signal over a frequency f range of interest. Here, we introduce a geometry factor for $1/f$ -dominated noise compatible with Hooge's relation [1] and derive a SNR geometry factor for arbitrarily shaped planar four-terminal devices. This framework is used to identify the SNR-optimal geometry of disk-, square-, and cross-shaped Hall sensors, cf. Fig. 1.

Theory

For planar Hall sensors of in-plane area A , homogenous planar charge carrier density n_{2D} and mobility in the conductive sensor region Ω , and operated in measurement modes a and b , cf. Fig. 2, at constant current I , the geometry dependence of the Hall voltage in mode a is given by the voltage-related Hall geometry factor [2]

$$G_{H,V} = \frac{\int_{\Omega} (\mathbf{i}_{a0} \times \mathbf{i}_{b0})_z dA}{\int_{\Omega} \mathbf{i}_{a0} \cdot \mathbf{i}_{a0} dA}, \quad (1)$$

where $\mathbf{i}_{a0}$ and $\mathbf{i}_{b0}$ denote the normalized in-plane line current densities in the two measurement modes at zero magnetic field, and $(\mathbf{i}_{a0} \times \mathbf{i}_{b0})_z$ is the z component of their cross product.

Analogously, a voltage-related geometry factor for $1/f$ -dominated noise can be derived as

$$G_{1/f,V} = \frac{A \int_{\Omega} (\mathbf{i}_{a0} \cdot \mathbf{i}_{b0})^2 dA}{\left(\int_{\Omega} \mathbf{i}_{a0} \cdot \mathbf{i}_{a0} dA \right)^2}. \quad (2)$$

A geometry dependence of $1/f$ -dominated output signal noise in terms of an effective area was

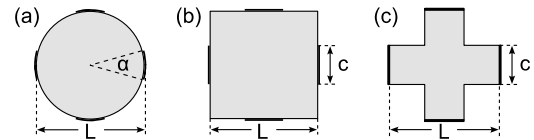


Fig. 1. Hall sensor geometries of characteristic dimension L : (a) Disk with contact opening angle α , (b) square, and (c) cross geometry with aspect ratio c/L .

first discussed in [3]. Here, this dependence is cast into the dimensionless voltage-related factor $G_{1/f,V}$. This formulation allows a straightforward generalization of the empirical Hooge relation [1]. In fact, the normalized voltage power spectral density (PSD) becomes

$$\frac{S_V}{V^2} = \frac{\alpha_{1/f}}{Nf} G_{1/f,V} = \frac{\alpha_{1/f}}{n_{2D}f} \frac{1}{L^2} \frac{1}{k_A^2} G_{1/f,V}, \quad (3)$$

where $\alpha_{1/f}$ is the Hooge factor, N the number of free charge carriers, and $k_A = \sqrt{A}/L$ a shape-dependent area-utilization factor. The standard Hooge relation is recovered for $G_{1/f,V} = 1$, holding for two-contact devices with homogeneous 1D current densities. Increasing the lateral sensor dimensions reduces $1/f$ noise at the expense of a larger footprint. The analysis is therefore restricted to a fixed characteristic dimension L of the structures, cf. Fig. 1. The resulting

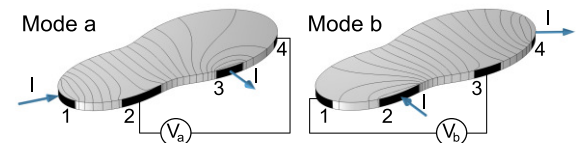


Fig. 2. Measurement configurations for modes a and b of an arbitrarily shaped planar four-terminal device operated with constant drive current I , yielding the output voltages V_a and V_b , respectively.

differences in the active sensor area are accounted for by k_A , listed in Tab. 1. Under this constraint, relying on the third expression in (3), we define the SNR geometry factor as

$$G_{SNR} = \frac{G_{H,V}}{\sqrt{G_{1/f,V}}} k_A = \frac{\int_{\Omega} (\mathbf{i}_{a0} \times \mathbf{i}_{b0})_z dA}{\sqrt{A \int_{\Omega} (\mathbf{i}_{a0} \cdot \mathbf{i}_{b0})^2 dA}} k_A. \quad (4)$$

It enables the direct comparison between different sensor shapes of identical L .

Numerical Simulation and Experiment

Finite-element simulations of $G_{H,V}$ and $G_{1/f,V}$ were performed using COMSOL, yielding G_{SNR} . The simulation results were supported by measurements on cross-shaped samples of GaAs Hall sensor test structures, featuring either varied aspect ratios c/L or uniform scaling at constant $c/L \approx 0.58$, cf. Fig. 1 (c), in a temperature-controlled setup at $T = 25^\circ\text{C}$. $G_{H,V}$ was obtained from van-der-Pauw measurements [4], and $G_{1/f,V}$ from PSD measurements at $V = 1\text{ V}$ for all specimens, evaluated at $f = 1\text{ kHz}$ after subtraction of the white-noise spectral background. The active sensor area A was estimated from the device layout. The Hooge factor was taken as $\alpha_{1/f} = 3 \times 10^{-4}$, based on its reported mobility dependence [1].

Results

As shown in Fig. 3, the investigated sensor geometries reveal a clear trade-off between signal and noise performance. With increasing design parameter ξ , defined as $\xi = \alpha$ for disks and $\xi = c/L$ for square and cross sensors, $G_{1/f,V}$ increases monotonically, whereas $G_{H,V}$ reaches a maximum at intermediate ξ . In general, the SNR-optimal sensor geometry does not coincide with the optimal Hall signal geometry. For all three shapes, $G_{1/f,V}$ increases strongly at large ξ values, since the current flow in both modes becomes more aligned, particularly between adjacent contacts, cf. Fig. 4. The $G_{H,V}$ values extracted from measurements and shown in Fig. 3 (c) are in very good agreement with the simulations. The extracted $G_{1/f,V}$ values reproduce the simulated trend well. However, their absolute value depends on $\alpha_{1/f}$ taken from literature [1] and may therefore be subject to systematic scaling. As expected, measurements on uniformly scaled crosses confirm that the geometry factors

Tab. 1: Area-utilization factor and optimal SNR values.

	Disk	Square	Cross
k_A (-)	$\sqrt{\pi}/2$	1	$\sqrt{2c/L - (c/L)^2}$
Max. G_{SNR}	0.56	0.52	0.59
ξ_{opt}	2.87°	0.07	0.58

are invariant under scaling. The optimal SNR values extracted from the simulation data are listed in Tab. 1. Crosses show the best SNR performance with coinciding Hall-signal- and SNR-optimal geometries. Disks outperform squares. For both, the SNR optimum is found at small ξ , differing from the Hall-signal-optimal geometry.

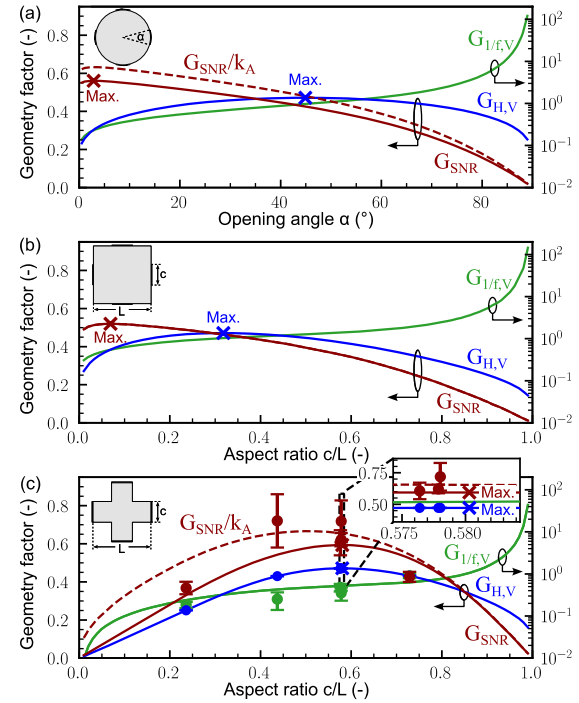


Fig. 3. Simulated geometry factors for (a) disk, (b) square, and (c) cross geometries. The blue and red x-symbols mark the maximum $G_{H,V}$ and G_{SNR} , respectively. For the cross geometries, the simulations are supported by measurements.

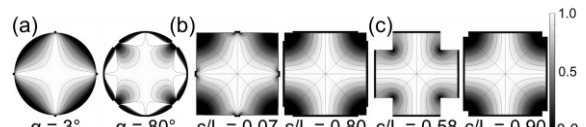


Fig. 4. Simulated spatial distributions of $k_A (\mathbf{i}_{a0} \times \mathbf{i}_{b0})_z / \sqrt{A (\mathbf{i}_{a0} \cdot \mathbf{i}_{b0})^2}$ for pairs of different (a) disk, (b) square, and (c) cross geometries.

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