

Improved Angle Sensing by Shape-from-Motion Calibration of 3D Magnetic Sensors

Bibhu Kar, Per Rutquist, Oliver Paul

*Laboratory for Microsystem Materials, IMTEK, University of Freiburg, Germany
bibhu.kar@imtek.uni-freiburg.de*

Summary:

Axis non-orthogonality in multi-axis magnetic sensors introduces cross-talk, distorts the reconstructed magnetic field, and limits measurement accuracy. In this contribution, we present a shape-from-motion-based method to estimate and compensate for axis non-orthogonality in a 3D magnetic sensor by exposing the sensor to a magnetic field of constant magnitude at six orientations. The key advantage of this approach is that it does not require precise control of the field directions nor the magnitude. In the reported case, we demonstrate the improvement of the measured relative angles by a factor of 3.5.

Keywords: Magnetic sensors, sensor calibration, axis non-orthogonality, inverse calibration, Shape-from-motion method

Introduction

In order to reconstruct a magnetic field reliably, accurate calibration of multi-axis magnetometers is essential. In practice, fabrication tolerances and assembly imperfections lead to gain mismatch, offset, and non-orthogonality between the sensor axes.

Conventional calibration of magnetic sensors requires an application of known magnetic fields in well-defined orientations, typically using Helmholtz coil systems. However, coil setups are prone to their own imperfections. To overcome these limitations, we implement a calibration method based on the shape-from-motion (SfM) method. SfM was originally developed in computer vision to recover 3D structures from multiple observations [1], and was later applied by Kim et al. [2] for the calibration of 2D MEMS force sensors.

SfM exploits the structure of the measured data acquired at different orientations, allowing the simultaneous estimation of sensor response parameters. In contrast to [2], in this work, we extend SfM to a 3D magnetic sensor. Furthermore, we demonstrate the capability of the method to improve angle sensing, even if the magnitude of the calibration magnetic field is not accurately known.

Method: Shape-from-motion calibration

A 3D magnetic sensor provides a measurement vector $z \in \mathbb{R}^3$, in response to a magnetic field vector $m \in \mathbb{R}^3$. Assuming a linear model, the relationship between z and m is expressed using a sensor response matrix S as:

$$z = mS. \quad (1)$$

For N measurements acquired under different sensor orientations in a magnetic field of constant magnitude, the model can be written in matrix form as:

$$Z = MS, \quad (2)$$

where $Z \in \mathbb{R}^{N \times 3}$ contains the measurement data and $M \in \mathbb{R}^{N \times 3}$ contains the corresponding field vectors.

Nevertheless, the matrix Z can be factorized using singular value decomposition (SVD) as:

$$Z \approx \hat{M}\hat{S}. \quad (3)$$

This factorization is not unique and, for non-collinear measurement orientations, defined up to an unknown invertible linear transformation. This ambiguity is resolved by exploiting the constant magnitude of the applied magnetic field, which introduces a quadratic constraint on the reconstructed vectors and enables the recovery of the sensor response matrix. A detailed explanation is available in [2].

In this contribution, only six measurement orientations are used for 3D calibration. These are sufficient to estimate the sensor response matrix S up to an unknown rotation and, if the calibration field magnitude is inaccurately known, up to a scaling factor.

The calibration matrix $C = S^{-1}$ is then used to reconstruct the applied magnetic field vectors from measured data. Further, we evaluate the axis non-orthogonality by computing the angles

between the effective sensing directions derived from C .

Measurements

For validating the method, we used a Bosch BMM350 sensor. This 3D magnetometer is mounted at the center of a Helmholtz coil setup generating a quasi-uniform magnetic field. The sensor was rotated using a stepper motor to acquire measurements under different orientations. To ensure sufficient excitation of all three sensor axes, we rotated the sensor around two different axes, as shown in Fig. 1.

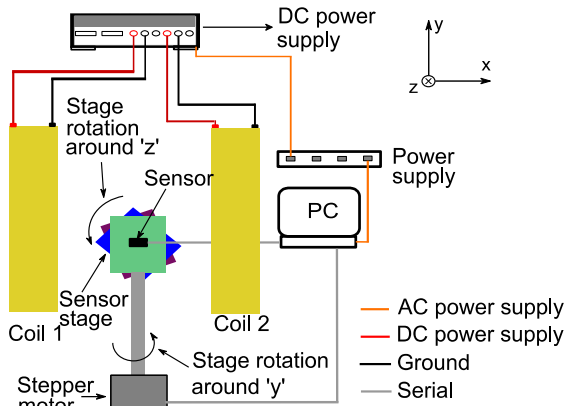


Fig. 1. Helmholtz coil setup with a rotational stage providing 2 DOF for sensor orientation.

Results

We measure the magnetic field components B_x , B_y , and B_z at six sensor orientations under a constant field. Fig. 2 shows the measured data and the reconstructed field vectors after SfM-based calibration.

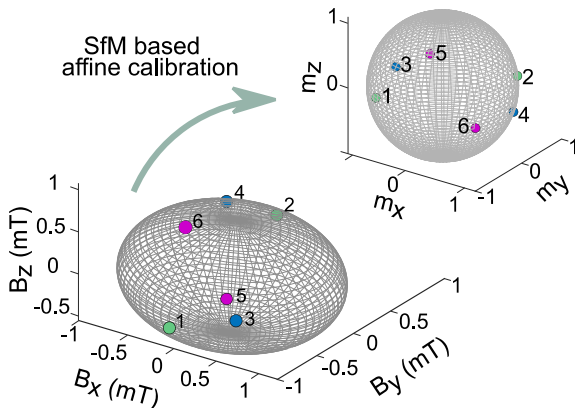


Fig. 2. Raw magnetic field vectors in six orientations and after SfM-based affine calibration. For visualization, the non-sphericity of the ellipsoid is exaggerated by a factor of 25.

The measurements form a distorted ellipsoid due to gain mismatch and axis misalignment. The SfM calibration reconstructs vectors that lie close to a unit sphere, indicating effective compensation of these distortions.

To validate the model, the sensor is rotated over 360° in three different planes, yielding 72 measurements per plane. Fig. 3 shows the angular deviation as a function of the reference angle for raw and SfM-calibrated data.

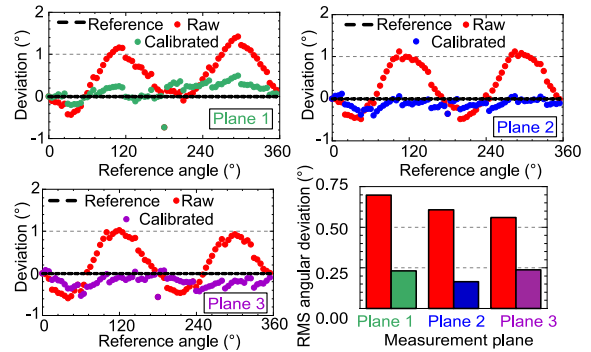


Fig. 3. Angular deviation vs. reference angle (raw vs. SfM-calibrated) for three planes (72 points each); bottom-right: RMS angular deviation before and after calibration.

SfM reconstructs the magnetic field vectors on a unit sphere; the remaining arbitrary rotation does not affect the determination of relative angles, as relative angles are rotation invariant [3]. The effective sensing directions are obtained directly using C and the angles between them are computed as:

$$\alpha_{ij} = \cos^{-1} \frac{m_i \cdot m_j}{|m_i| \cdot |m_j|}. \quad (4)$$

Tab. 1 summarizes the resulting inter-axis angles. All angles are close to 90° , with deviations below 0.6° . Further, the uncertainty estimated from repeated measurements is around 0.1° , indicating that the observed non-orthogonality is small but statistically significant.

Tab. 1: Angles between pairs of sensor axes, showing minor deviations from 90°

Axis pair	x-y	x-z	y-z
Angle α ($^\circ$)	89.85 ± 0.09	89.48 ± 0.10	90.49 ± 0.06

References

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