

Study of Width-over-length Aspect Ratio on Mode Ordering for 2D Nanomechanical Resonant Sensors

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Summary: Two-dimensional (2D) nanoelectromechanical system (NEMS) resonators, which offer high sensitivity, show strong potential for sensing applications. Among them, the fundamental mode is typically used as the operating mode for detecting external physical quantities such as pressure, mass, and force. Operating in this mode maximizes the sensor's displacement amplitude, which translates to a stronger output signal and enhanced overall sensitivity. However, a comprehensive and systematic understanding of how to identify the working mode is still lacking. In this work, we analyze how the width-to-length aspect ratio affects the ordering of resonant modes in doubly clamped 2D MoS₂ NEMS resonators. Through finite element method (FEM) simulation, the longitudinal fundamental mode remains largely insensitive to device width, whereas the eigenfrequencies of transverse modes vary strongly with aspect ratio. This leads to clarify mode-crossing events and a reordering of the overall mode sequence.

Keywords: Two-Dimensional materials, Nanomechanical resonant sensors, Eigenmodes, Width-over-length aspect ratio, FEM simulation

Background, Motivation and Objective

NEMS resonators are ideal ultrasensitive sensors due to their extremely small size, high frequency and low power consumption [1]. Physical perturbations such as mass adsorption (Fig. 1 (a)) or temperature changes, measurably shift the fundamental resonance peak (Fig. 1 (b) and (c)). However, reliable characterization requires a clear distinction of the working mode shapes. This work provides a comprehensive study how to differentiate these modes by FEM simulation as width-over-length ratio varies.

Simulation Setup and Theoretical Model

In our study, we model a doubly clamped MoS₂ membrane, a prevalent material in 2D NEMS (Fig. 1 (a)). Key properties include length (l), width (w), width-over-length aspect ratio ($a = w/l$), thickness (t), i^{th} resonance frequency (f_i). For highly narrow devices ($a \ll 1$), the membrane behaves like an ideal one-dimensional beam with predictable frequencies [2][3]:

$$f_i = \frac{\lambda_i^2 \cdot t}{2\pi \cdot l^2 \cdot \sqrt{12}} \cdot \sqrt{\frac{E}{\rho \cdot (1 - \nu)}} \quad (1)$$

where E is Young's modulus, ρ is density, ν is Poisson's ratio and λ_i are solutions from the characteristic equation of doubly clamped boundary condition $\cos(\lambda_i)\cosh(\lambda_i) = 1$. This lead to values of $\lambda_1 = 4.73, \lambda_2 = 7.85$, etc.

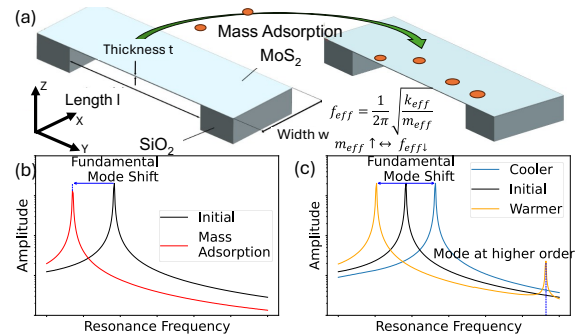


Fig. 1. (a) Schematic of device structure and mass adsorption. (b) Illustration of the frequency shift due to mass adsorption (c) Impact to temperature change

Tab. 1: Theoretical frequency of example device

Properties [4]	Result	
$l = 2\mu m$	Mode	Eigenfrequency
$t = 10nm$	Fundamental	19.40 MHz
$E = 270GPa$		
$\nu = 0.23$	Secondary	53.49 MHz
$\rho = 5000kg/m^3$		

Through calculation based on Tab. 1, the 1st and 2nd modes of an example device are 19.40 MHz and 53.49 MHz, respectively. However, as w increases, the 1D approximation might fail to cover all. To identify the fundamental mode and classify the 2D oscillation of the membrane, we introduce the notation (m, n) , where m describe the mode order along the free direction and n the

clamped direction. The spatial mode shape (x, y) (Fig. 1(a)) and frequency are [5]:

$$(x, y) = k \cdot \cos\left(\frac{m \cdot x \cdot \pi}{w}\right) \cdot \sin\left(\frac{n \cdot y \cdot \pi}{l}\right) \quad (2)$$

$$f_{mn} \propto \sqrt{\left(\frac{m}{w}\right)^2 + \left(\frac{n}{l}\right)^2} = \sqrt{\left(\frac{m}{a \cdot l}\right)^2 + \left(\frac{n}{l}\right)^2} \quad (3)$$

where k acts as a scaling constant.

Results and Discussion

For a purely clamped mode $(0, n)$ where no transversal bending occurs (Fig. 2), the approximation in Tab. 1 remains highly valid, respectively, regardless of the increasing width.

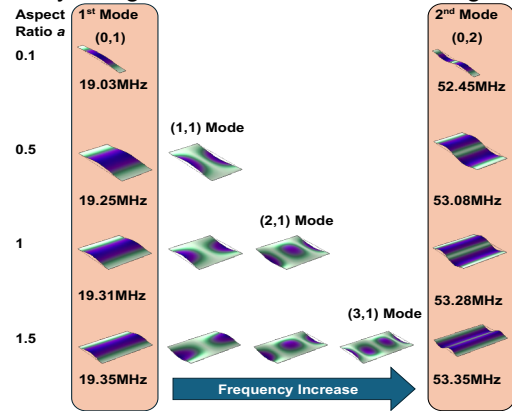


Fig. 2. Demonstration of 2D mode order varying with aspect ratio.

Complex transversal modes m begin to emerge when the aspect ratio a exceeds $a > 0.1$. In the intermediate aspect ratio $0.3 < a \leq 1$, the frequencies of transverse modes spread widely across 100MHz (Fig. 3). As an example, it can be observed that mode(1,1) cross mode (0,2) at $a \approx 0.25$ (Fig. 3 (a)). Because the frequencies of higher order (m, n) modes asymptotically converge toward the frequencies of the fundamental $(0, n)$ modes (Fig. 3(b)), it remains reasonable to use the $(0, n)$ approximation for device with an exceptionally high aspect ratio ($a \gg 10$).

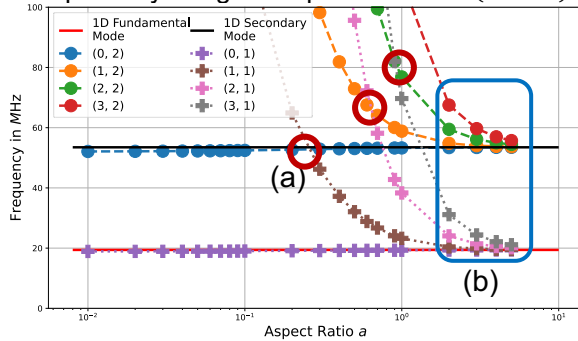


Fig. 3. Modeled frequencies of various (m, n) modes against the aspect ratio. (a) Mode-crossing. (b) Convergence

To experimentally validate these findings, we use an RF down-mixing detection scheme to

measure the resonance of doubly clamped 2D MoS₂ resonators [6], detecting the same frequency shift shown by the simulations (Fig. 4).

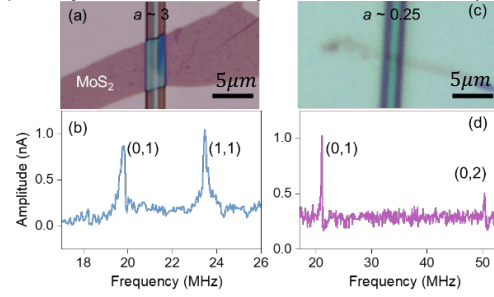


Fig. 4. (a)(b) Measured modes of a device with $a \sim 3$. (c)(d) Measured modes of a device with $a \sim 0.25$. Reprinted in part with permission from ref [7]. Copyright 2026 IEEE.

Conclusion

In summary, this study shows the mode-crossing induced by the width-over-length aspect ratio in 2D MoS₂ resonant sensors. Highly accurate sensing can be achieved at a strongly asymmetric width-over-length geometry, either $a \ll 1$ to isolate the frequencies, or $a \gg 10$ to merge the non-fundamental modes to fundamental ones.

Acknowledgments

We thank Ines Kraiem for fabricating the receiving substrate.

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