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Flexible Equivalent Circuit Modeling for Piezoelectric Vibration Energy Harvesters

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Summary:
We present a parameter extraction method for piezoelectrically coupled equivalent circuit models. We describe a method for modeling multi electrode configurations, that can be configured as needed by introducing additional coupled capacitors.

Keywords: Piezoelectric Energy Harvesting, Electromechanical Modeling, Finite Element Method, Equivalent Circuit Models, Multi Electrode Configurations

Introduction

When designing Vibration Energy Harvesting devices (VEHs), both the electromechanical structure as well as the electronic circuit have to be optimized. A common approach in describing piezoelectric VEHs is a multi-modal electromechanical model, incorporating coupling between mechanical eigenmodes and an electric capacitance, and vice-versa. When pursuing a system simulation approach, a modal model may be implemented as an Equivalent Circuit Model (ECM) in an electrical circuit simulator. In [1] such multi-modal models are examined in detail and the importance of distributed mechanical parameters, as well as proper electromechanical coupling is underlined. Moreover, we have proposed a modal reduction technique, based on Finite Element (FE) modal analysis, resulting in a reduced order coupled system [2]. In [3], a FE based approach is presented to extract the Parameters for an ECM from various frequency-resolved simulations of the electric admittance of a VEH.

In this work, we extend our approach [2] to parameter extraction for electrical equivalent circuit models and propose a circuit structure for multi-electrode harvesters.

Methods

The basic structure of our ECM is depicted in Fig. 1. The non-greyed part is a coupled model as used by, e.g, [4], [3]. The equivalent elements L_n, C_n, R_n represent a mechanical mode, the current is a mechanical displacement in generalized coordinates. The ideal transformer with winding-ratio N_n couples mechanical displacement with electric current through the static capacitance C_0 and vice-versa.

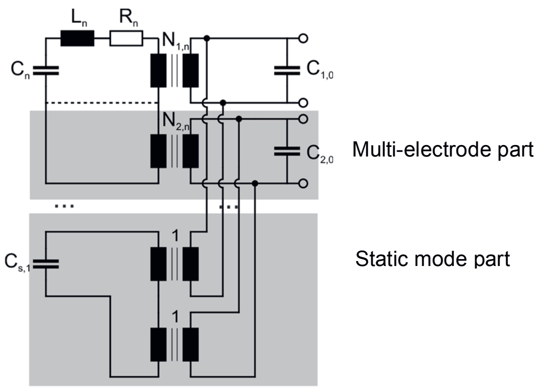

Fig. 1. Strucure of the equivalent circuit model.

Considering multi-electrode structures, we add additional transformers and static capacitors. As an example we use two configurations of a trimorph cantilever, depicted in Fig. 2. One configuration is completely symmetrical. In a second configuration the lower piezoelectric layer is shorter than the upper layer.

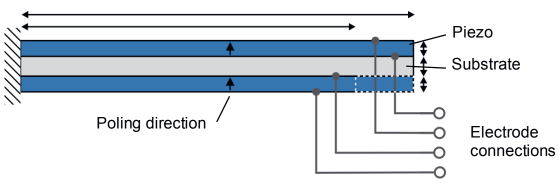

Fig. 2. Trimorph Energy Harvester, used here as an example.

Parameters R_n, L_n, C_n are derived by solving a generalized eigenvalue problem for a number of M eigenmodes, for a FE model with all electrodes in short circuit condition and subsequent diagonalization of the FE matrices. As we pointed out in [2], it is important to include the piezoelectric effect already at this stage of the mechanical eigenmode calculation, as the electromechanical coupling exerts material stresses even in external short circuit conditions. The coupling (i.e., winding ratios N_n) are computed by imposing the eigenvectors (i.e., mode shapes and corresponding electric potentials) onto the original FE stiffness matrix. The static Capacitances C_0 are derived from static computations, yielding the unclamped capacitances C^t. As the electromechanical capacitors C_n will transform to the electrode through the ideal transformers by $C_{el} = N^2 C_n$, the capacitances from all considered modes must be subtracted [3]

$$C_0 = C^t - \sum C_{el} \quad (1)$$

The static capacitance of each of the electrodes therefore coincides with the result from a static computation of the original FE model, regardless of the number of modes considered. If we, however, want to investigate different interconnections between electrodes, e.g., series or parallel connections, the different static bending shapes in different electrical configurations will change the static capacitances that are being observed. As an example, the static, unclamped capacitance of the symmetrical trimorph from Fig. 2 in parallel configuration was 6% larger than the sum of the two single capacitances. This effect can be captured by considering a sufficiently large number of electromechanical mode shapes, that may well exceed the frequency range of interest. Neglecting this effect, however, will result in deviations in both resonance frequency as well as in magnitude of the electrical impedance.

With a view to resolving this trade-off between necessary number modes and precision, we propose the introduction of *static modes* that are represented in the ECM by capacitors that couple via ideal transformers to the static capacitances (cf. Fig. 1). Each additional capacitance is derived from a static computation of a possible electric configuration. In case of the two-electrode example (cf. Fig. 2), the static modes are “bending” when operated in anti-parallel or anti-serial configuration, and “elongation” when operated in parallel or serial configuration. Additionally, each electrode capacitance C_0 is now computed as the clamped capacitance C^s. The coupled capacitors are then calculated similarly to (1), depending on the sign of the respective winding ratios. In doing so, effects on the static capacitance and therefore the overall electric impedance are captured without considering an unnecessarily high number of eigenmodes.

Results

Figure 3 shows the electric impedance around the first bending mode frequency of the symmetrical trimorph model with anti-parallel configuration. The 4 mode ECM shows a considerable deviation due to an incorrect static capacitance. When considering 20 eigenmodes, the ECM coincides perfectly with the original FE model. The same is achieved, however, if the 4 mode ECM is extended by the static mode circuits as proposed here.

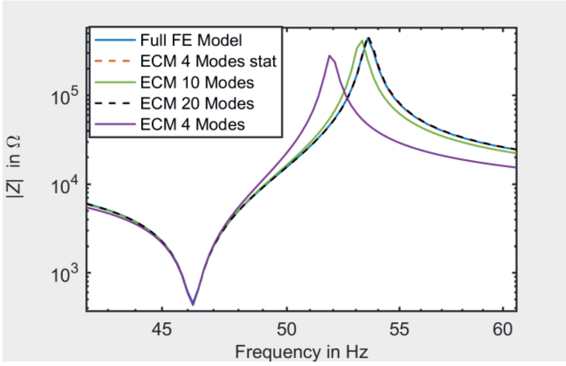

Fig. 3 Anti-parallel configuration, 1st bending mode.

For the unsymmetrical variation from Fig. 2, a parallel connection of the electrodes is shown in Fig. 5. The uncompensated 4 mode ECM shows a much lower effective capacitance, resulting in considerable deviations from the original model.

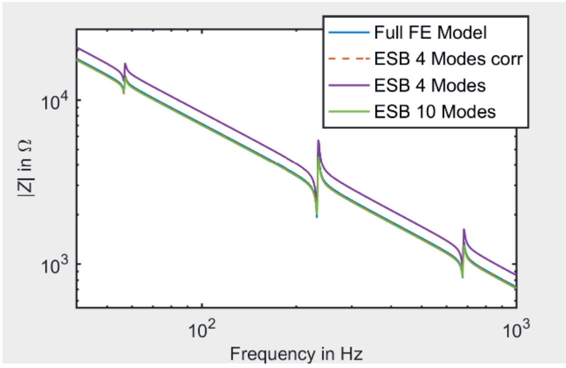

Fig. 5. Non-symmetrical trimorph (Fig. 2) in parallel-configuration.

References

[1] A. Erturk, D.J. Inman, Issues in mathematical modeling of piezoelectric energy harvesters, *Smart Materials and Structures 17,* 065016 (2008), doi: 10.1088/0964-1726/17/6/065016

[2] D. Gedeon, S.J. Rupitsch, Finite element based system simulation for piezoelectric vibration energy harvesters, *Intelligent Material Systems and Structures 29,* 1333–1347, doi: 10.1177/1045389X17733328

[3] Y. Yang, L. Tang, Equivalent circuit modeling of piezoelectric energy harvesters, *Intelligent Material Systems and Structures 20,* 2223-2235 (2009); doi: 10.1177/1045389X09351757

[4] N.G. Elvin, A.A. Elvin, A general equivalent circuit model for piezoelectric generators, *Intelligent Material Systems and Structures 20*, 3-9, doi: 10.1177/1045389X08089957