

Modeling the fluid-structure interaction of non-conventional vibrational modes for MEMS fluid sensing

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Summary:

Understanding of the fluid-structure interaction between a mechanical resonator and a surrounding fluid is key for understanding and predicting the performance of MEMS fluid property sensors. Here, we present a novel method for modelling the fluid-structure interaction between a viscous fluid and vibrational eigenmodes of a plate-like MEMS resonator. The elastic dynamics of the MEMS resonator numerically determined by a finite element method while the fluid flow is obtained from a boundary integral formulation. With this method we compute the spectral response of MEMS plate resonators in fluids.

Keywords: fluid sensing, MEMS resonators, plates, fluid-structure interaction, simulation

Background, Motivation and Objective

The characterization of fluid properties like density or viscosity is a focal area in fluid sensing. Fluid sensors based on micromechanical systems (MEMS) have the potential for widespread use in various applications, like the monitoring of technical fluids like motor oil or medical diagnosis in lab-on-chip systems, due to their low-production cost and high integrability in complex sensor systems. In all applications, the measurement of fluid properties requires an interaction between the sensor and the fluid environment. Such an interaction is established by exciting a vibrational eigenmode of a MEMS resonator. Often, sensors have simple geometries like cantilever beam structures. Such resonators are relatively simple to fabricate and their vibrational eigenmodes are readily modeled with Euler Bernoulli beam theory. A typical vibrational eigenmode of a beam resonator is shown in figure 1a. However, the quality factor of beam-like MEMS resonators is usually very low in liquids which implies that reliable measurements of fluid properties is often difficult or even not feasible, especially in highly viscous fluids. A possible solution to this problem is the use of vibrational modes which have are commonly not considered for fluid sensing. An example of such a non-conventional mode is depicted in figure 1b. These non-conventional vibrational modes exhibit extraordinary high quality factors even in highly viscous fluids [1, 2] which allows for measurements in highly viscous fluids. However, the physical reason for the high quality factors of non-conventional modes in fluids remains elusive. Here, we introduce a novel method for

modelling the fluid-structure interaction between a fluid and plate-like MEMS resonators.

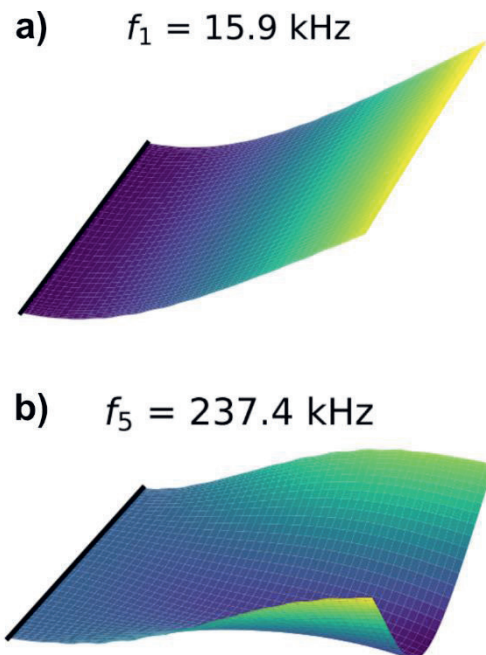


Fig. 1. Numerical simulation of vibrational eigenmodes of a cantilevered plate resonator clamped at the right edge (marked by thick black line). The mode in (a) is also observed in one-dimensional beam structures whereas the mode in (b) can only be found in two-dimensional plate structures.

Description of the Method

Two components are required for modelling the fluid-structure interaction between a fluid and plate resonator: the elastic dynamics of plate and the fluid flow. While beam resonators are readily

described by the Euler Bernoulli equation, modelling the dynamics of plate resonators is based on the Kirchhoff Love equation. In contrast to the Euler Bernoulli equation, analytic solutions for the Kirchhoff Love equation exist only for special cases. Therefore, we use a finite element method to obtain numerical solutions of the plate dynamics. The method weakly imposes the physical continuity requirements to the solution by introducing a penalty term which allows for the use of standard Lagrange-type elements.

We focus on MEMS resonators for which the characteristic length scale of the fluid flow is given by the width of the clamped side. This side has a typical width of 100 to 1000 μm . With this assumption the fluid flow can be modelled with the Stokes equation for an incompressible viscous fluid. We employ a stream function description of the flow and express the problem as a boundary integral equation which we solve numerically. From the resulting fluid flow as depicted in figure 2 we determine the hydrodynamic force on the plate resonator.

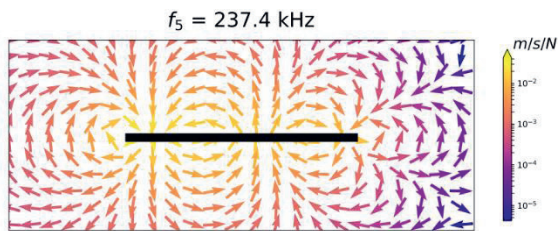


Fig. 2. Fluid flow around the cross section of the resonator for the mode shown in figure 1b. The flow velocity is normalized to the applied drive force.

Using this approach, we are able to determine for the first time the spectral response of two-dimensional MEMS resonators which has not been possible before with established theory for one-dimensional beam-like resonators [3].

Results

We apply the proposed method to a cantilevered plate resonator with a size of $300 \times 300 \times 5 \mu\text{m}^3$ immersed in water. The plate is excited at one of

its free corners and we solve for the spectrum directly in the Fourier domain. The resulting spectral response is shown in figure 3 (blue line). To compare our results with existing theory we also plot the corresponding results based on Euler Bernoulli beam theory. The peak at the lowest frequency corresponds to a flexural mode which can also be found in one-dimensional beam resonators. Therefore, both curves coincide. The second peak is a torsional mode not described by Euler Bernoulli beam theory. The peak at 100 kHz corresponds to the second beam mode and consequently both theories agree with each other. The mode at 237 kHz is the mode shown in figure 1b and its response can only be predicted with the proposed method.

Conclusion

We present a method for modelling the fluid-structure interaction of MEMS plate resonators. With this method we are able to predict the spectral response of plate resonators in liquids. These spectra pave the way for quantitative measurements of fluid properties and novel designs of fluid sensors which go beyond one-dimensional geometries.

References

- [1] Kucera, M. et al. Characterisation of multi roof tile-shaped out-of-plane vibrational modes in aluminium-nitride-actuated self-sensing micro-resonators in liquid media. *Appl. Phys. Lett.* 107, 053506 (2015).
- [2] Pfusterschmied, G. et al. Temperature dependent performance of piezoelectric MEMS resonators for viscosity and density determination of liquids. *J. Micromechanics Microengineering* 25, 105014 (2015).
- [3] Sader, J. E. Frequency response of cantilever beams immersed in viscous fluids with applications to the atomic force microscope. *J. Appl. Phys.* 84, 64–76 (1998).

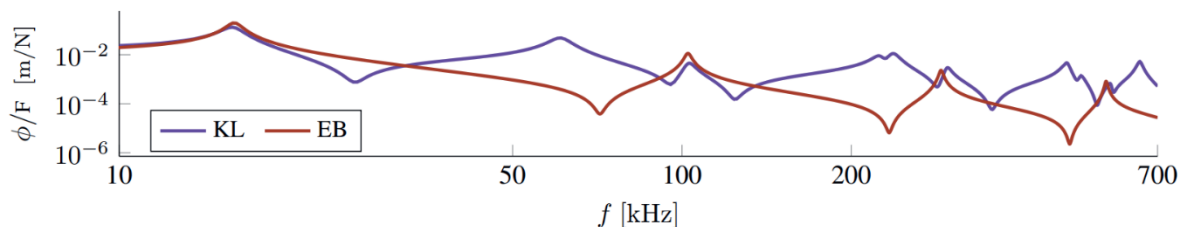


Fig. 3. Simulated spectral response of a plate resonator immerse in water. The blue line is based on Kirchhoff Love plate theory while the red line is based on Euler Bernoulli beam theory. While both theories predict the resonances of modes that can be found both in beams and in plates. The Euler Bernoulli based theory fails to predict resonance of modes which can only be found in plates.