

# Measurement Uncertainty Assessment for Virtual Assembly

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## Summary:

In dimensional metrology, a datum system is used for defining a coordinate system to enable the evaluation of geometrical tolerances of workpieces. With regard to function-oriented tolerancing representing the workpiece's function optimally, the physical workpiece contact has to be recreated by the datum system. Therefore, a new registration approach is used, where contact points of the acquired measurement point clouds determine the datums. In this paper, the propagation of the measurement uncertainty of contact points towards the registration result is discussed, having an impact on derived measurands.

**Keywords:** Virtual Assembly, Datum Definition, Geometrical Tolerancing, Dimensional Measurement, Uncertainty Assessment

## Trends in Production and Metrology

Higher product requirements and smaller tolerances trigger an increasing attention on the geometrical assurance in manufacturing processes. A paradigm shift in the ISO system for Geometrical Product Specifications (ISO GPS) is the introduction of the Skin Model Shapes (SMS), where the part geometry is described by a holistic, discrete surface representation. By applying SMS, mainly the description of form tolerances could be improved. While deviations of size and location have decreased ten times every 50 years, form deviations remain at nearly constant level, and thus become an increasingly relevant field of research. Optical metrology systems and Computed Tomography (CT) enable the generation of SMS by capturing large measurement point sets (point clouds) in short acquisition time [1, 3].

## Current method of datum definition

The main purpose of a datum system is to define a common coordinate system for measured point clouds of one or more workpieces. Tolerance zones are aligned according to the datum system, thus the derived tolerance value is sensitive to the datum system definition. The recent method for datum system definition is described in ISO 5459:2011 [2]. Here, the datum system is defined by approximated regular geometries like tangential planes (see Figure 1). However, it is disadvantageous that local form deviations are not considered, since approximated geometrical elements with ideal forms are employed [2, 3].

## Method of Virtual Assembly

By the novel method of Virtual Assembly (VA), the holistic surface information is used for the datum definition. As shown in Figure 1 (right), surface 2 is aligned relatively to surface 1 by the minimization of their distances, avoiding a surface intersection. The registration of the datum is mathematically stated as an optimization problem [4].

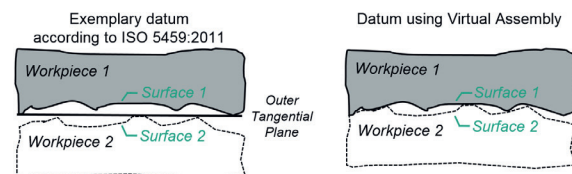


Fig. 1. Exemplary datum systems based on current ISO definition (left) and according to VA approach (right)

The signed Euclidean distance  $d_{s,n}$  of  $N$  corresponding pairs of points  $p_{1,n}$  and  $p_{2,n}$ ,  $n = 1 \dots N$ , from point set  $P_1$  of surface 1 and point set  $P_2$  of surface 2 is used to compute the objective function (1).

$$f(T_x, T_y, T_z, \phi, \theta, \psi) = \sum_{n=1}^N d_{s,n}^2 = \min! \quad (1)$$

Here, the optimization variables  $T_x$ ,  $T_y$  and  $T_z$  determine the translation and  $\phi$ ,  $\theta$ ,  $\psi$  are the Euler angles of the rigid transformation of  $P_2$  to  $P_1$ . The avoidance of surface intersection can be either formulated as a constraint, allowing  $d_{s,n} \geq 0$  only, or by introducing a penalty term as summand to  $f$  in order to penalize intersection.

### Concept for Uncertainty Assessment

A complete measurement result includes the associated measurement uncertainty. For the VA approach, the uncertainties of individual contact points of the acquired surfaces have a strong impact on the datum system. Hence, derived extrinsic measurands such as sizes or position tolerances are influenced by the uncertainty of the contact points. In this paper, the uncertainty is experimentally determined by *Type A* evaluation according to the *Guide to the Expression of Uncertainty* (GUM) [5]. The uncertainties  $u_{q,n}$  considering the  $n$ -th point  $p_{q,n,m}$  of a surface  $q$  and of repetition  $m \in [1; M]$  are estimated according to (2) with  $\bar{p}_{q,n}$  as mean value of  $M$  repetitions of  $p_{q,n}$ . In context of the VA approach, always two surfaces are registered, so that  $q \in [1; 2] \mid q \in \mathbb{N}$ .

$$u_{q,n} = \sqrt{\frac{1}{M-1} \sum_{m=1}^M (p_{q,n,m} - \bar{p}_{q,n})^2} \quad (2)$$

The general measurement model is described by  $q$  input estimates  $x_q$  and measurand  $y = f(x_q)$  as output. Thus, the uncertainty of the measurand contains contributors of  $q$  input sizes [5]. Here, the uncertainty  $u_1$  of surface 1 and the composed uncertainty  $u_{2,c}$  of surface 2 are considered in the combined standard uncertainty  $u_c$  (3) of the measurement procedure [6]. Because the transformation uncertainty  $u_T$  propagates to uncertainty  $u_2$  of the point cloud to register, the composed uncertainty  $u_{2,c}$  (4) is considered. The uncertainty  $u_{2,c}$  depends on the contact points determined by VA with their particular  $u_T$  and the uncertainty of the contact points in  $u_2$ , so that all points in the point set are affected by these uncertainties. In (3),  $\partial f / \partial x_q$  is the  $q$ -th sensitivity coefficient, equal to 1 here for all  $q$ , stated in [6].

$$u_c(y) = \sqrt{\left(\frac{\partial f}{\partial x_1}\right)^2 \cdot u_1^2 + \left(\frac{\partial f}{\partial x_{2,c}}\right)^2 \cdot u_{2,c}^2} \quad (3)$$

$$u_{2,c} = f(u_2, u_T) = f(u_2, u_x, u_y, u_z, u_\phi, u_\theta, u_\psi) \quad (4)$$

If the uncertainty  $u_c(y)$  is computed considering more than one pair of points  $\{p_{1,n}; p_{2,n}\}$ , the propagated uncertainties have to be merged in a certain distribution model, e.g. the Fisher-Bingham-Kent distribution (FBK) in [7]. FBK describes the distribution on a sphere. Assuming an isotropic uncertainty  $u_{q,n}$  here, meaning that  $u_{q,n}$  is independent of the probing direction, the uncertainty representation in 3D is a sphere. According to [5, 6], the expanded uncertainty  $U$  is stated as  $U = k \cdot u_c(y)$ , with  $k = 2$  as coverage factor.

### Nonlinear uncertainty propagation

The uncertainty assigned to a point in 3D space can be described by the  $3 \times 3$  covariance matrix. Due to orientation uncertainties  $u_\phi, u_\theta$  and  $u_\psi$ ,

the uncertainty propagation from the original to the transformed point cloud is nonlinear. Position uncertainties  $u_x, u_y$  and  $u_z$  propagate linearly. The propagation of orientation uncertainty considering the FBK distribution is described in [7].

### Case Study

The uncertainty propagation of a linear guide including rail and slider is evaluated as shown in Figure 2. The measurement uncertainties are acquired by 20 repeated CT scans, the transformation uncertainties by 20 repeated VA registrations, where the initial point cloud position is varied by random numbers from a normal distribution with  $\sigma_T = 0.05$  mm and the orientation is varied about  $\sigma_R = 0.002^\circ$ . Furtheron, the uncertainty  $U$  can be validated against the uncertainty acquired by repeated VAs. By doing so, the model for VA can be compared to the current ISO 5459 datum definition, allowing comparing uncertainty levels of both approaches quantitatively.

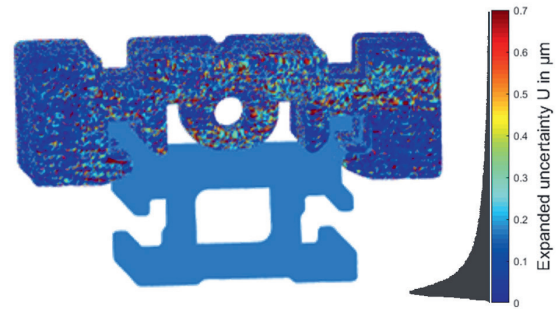


Fig. 2. Exemplarily propagated uncertainty  $U$  of the slider with respect to the rail including histogram of  $U$

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