

# 6 DoF Pose Estimation with $\mu\text{m}/\mu\text{rad}$ Accuracy Based on Laser Multilateration

Jan Nitsche<sup>1</sup>, Matthias Franke<sup>1</sup>, Nils Haverkamp<sup>2</sup>, Daniel Heißelmann<sup>1</sup>

<sup>1</sup> Physikalisch-Technische Bundesanstalt, Bundesallee 100, Braunschweig, Germany,

<sup>2</sup> Carl Zeiss Industrielle Messtechnik, Carl-Zeiss-Str. 22, Oberkochen, Germany

Jan.nitsche@ptb.de

## Summary:

Laser multilateration is a well-established technology for 3D coordinate measurements. An extended multilateration setup using seven tracking laser interferometers and a triple-retroreflector target is described, which allows for the estimation of a full six degree of freedom pose of the observed target. In an experimental setup with four degrees of freedom, the concept was proved and shows promising results in comparison with a precision coordinate measuring machine.

**Keywords:** multilateration, six degrees of freedom, self-calibration, pose estimation

## Introduction

The precise estimation of a six degree of freedom (6 DoF) pose (position plus orientation) of a given object in space plays an important role in robotics, automation and autonomous navigation. The pose of the end effector of industrial robots for example is essential for the precision of automated assembly processes. Different solutions for this problem are proposed, based on indoor gps systems [1], laser tracker measurements [2] or camera based sensors [3] and reach accuracies below 1 mm / 1 mrad.

Multilateration is a well-established technology to identify 3D coordinates of points in space [4,5]. Using laser interferometers for the distance measurement, accuracies of a few micrometers can be reached, depending on the size of the working space. By extending the multilateration principle from identifying 3D points to 6 DoF pose, higher precision of the pose estimation is expected.

## 6 DoF Multilateration

3D laser multilateration uses distances measured between three or more interferometric base stations and one retroreflector target to identify the 3D position of that target [5]. Results are achieved by minimizing the sum of squared residuals  $w_{ij}$ :

$$w_{ij} = \sqrt{(\Delta x_{ij})^2 + (\Delta y_{ij})^2 + (\Delta z_{ij})^2} - l_{ij} - l_{0j} \quad (1)$$

with  $i = 1, \dots, n$  as the number of target positions  $j = 1, \dots, m$  as the number of base stations,  $\Delta x_{ij}, \Delta y_{ij}, \Delta z_{ij}$  representing the coordinate differences between target position  $i$  and base station

$j$ .  $l_{ij}$  is the measured length change between target position  $i$  and base station  $j$  and  $l_{0j}$  is the dead path of each laser interferometer.

To extend the laser multilateration principle to identify a 6 DoF pose, an observed target requires three or more retroreflectors in a non-coplanar arrangement. The target is observed by at least 6 tracking laser interferometers in a non-coplanar setup. If the coordinates and dead paths of the interferometers need to be identified in a self-calibration process, at least seven interferometers are needed. Each retroreflector is observed by at least one interferometer. In addition to the residual functions (1), boundary conditions of the observed target are required:

$$w_{ik} = \sqrt{(\Delta x_{ik})^2 + (\Delta y_{ik})^2 + (\Delta z_{ik})^2} - l_k \quad (2)$$

with  $k = 1, \dots, p$  as any set of two retroreflectors,  $\Delta x_{ik}, \Delta y_{ik}, \Delta z_{ik}$  the coordinate differences between the two retroreflectors  $k$  and  $l_k$  the distance between the two retroreflectors. The resulting set of equations (1) and (2) contains  $n \cdot (m + p)$  equations,  $4 \cdot m$  unknowns for the interferometers,  $3 \cdot n \cdot p$  unknowns for the retroreflector coordinates and  $p$  unknowns for the retroreflector distances. For a minimum required setup of seven interferometers and three retroreflectors, this results in  $31 + 9 \cdot n$  unknowns and  $10 \cdot n$  equations. Using a dataset of at least 32 measurement points results in an overdetermined equation system which can be used for self-calibration of the unknown system parameters. It is important however that the dataset contains target positions with different orientations.

## Experimental Setup

An experimental setup was installed using a precision coordinate measuring machine (CMM) with additional rotational axis in the probing head. Three cat's eye retroreflectors are mounted to the probing head in a right-angled triangle with legs of 140 mm and 150 mm length. This setup provides a 4 DoF target. Seven tracking laser interferometers are installed along the short sides of the CMM working space in different  $z$  positions. For simultaneous data acquisition, six interferometers are triggered externally by one master interferometer. Reference positions of the CMM are acquired after a standstill period of 1 second. A grid of  $3 \times 3 \times 3$  target positions is recorded in a measurement volume of 820 mm x 550 mm x 80 mm. Each position is measured two times in three target orientations of  $0^\circ$ ,  $120^\circ$  and  $240^\circ$  respectively.

Data processing is performed in python. The overdetermined equation systems are solved using the Levenberg-Marquardt algorithm implemented in `scipy.optimize` package.

The limitation of the mover to 4 DoF requires a different approach for self-calibration as described in the previous section. In a first step, all seven interferometers are logged on to one retroreflector resulting in a classical 3D multilateration setup. This setup is used to calibrate the coordinates of the interferometers. In a second step the interferometers are split up to observe all three retroreflectors. In this step, dead paths of the interferometers and distances between the retroreflectors are calibrated.

## First Results

The previously described setup was used to calibrate the multilateration system. Due to the delicate surfaces of the retroreflectors it was not possible to calibrate the position of the retroreflectors in relation to the CMM coordinate system. The residuals of the optimized equation system however are below  $0.15 \mu\text{m}$ .

For further evaluation of the multilateration setup, seven additional target positions in one orientation along a straight line through the measurement volume were evaluated. For this evaluation, the difference between the spatial distance resulting from the CMM readings and the multilateration system is calculated. The maximum deviation between the two independent systems over a measurement length of 460 mm is below  $0.5 \mu\text{m}$ .

The equation system used for optimization is based on cartesian coordinates for each retroreflector. From the given distances between the retroreflectors and the coordinate deviations, angular deviations of the pose estimation can be calculated. In the given setup of a shorter leg of 140 mm and coordinate deviations of  $0.5 \mu\text{m}$  for each retroreflector, an angular deviation of  $7.1 \mu\text{rad}$  is expected.

## Conclusion

A multilateration setup for 6 DoF pose estimation using interferometric length measurement was developed and installed. In a reduced experimental setup with a 4 DoF positioning system, the setup could be implemented and calibrated. A first comparison to a precision CMM shows deviations of  $0.5 \mu\text{m}$  for a measurement length of 460 mm.

As a next step, full 6 DoF pose estimation using an appropriate positioning system will be analyzed. Further evaluation of the measurement results by comparison measurements and a measurement uncertainty evaluation will be performed.

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